Phys 6021: Homework I

due January 25, 2019

1. *[6 pts]* Show that the general solution of the free, time-independent Schrödinger equation

$$H_0|\Psi\rangle = E|\Psi\rangle$$

with $E = \hbar^2 k^2/(2m) > 0$ can be written as

$$\Psi_k(\vec{r}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm} j_l(kr) Y_{lm}(\theta, \varphi)$$

Determine the coefficients C_{lm}.

2. *[24 pts]* **Inelastic Scattering**

Inelastic scattering and reactions are included in the single-channel scattering formalism by viewing these processes as "absorbing" particles from an incident beam, where the absorption is described by a complex potential, also called optical potential,

$$V(r) = U(r) + iW(r) \tag{1}$$

(a) Derive the continuity equation for the time-dependent Schrödinger equation including a complex potential.

(b) Show that this leads to the relation

$$\frac{\partial}{\partial t} \int d^3r |\psi|^2 = 2 \int d^3r W(r) |\psi|^2 - \int d\mathbf{A} \cdot \frac{\psi^* \nabla \psi - \psi \nabla \psi^*}{2\mu i}. \tag{2}$$

(c) Show that $W(r)$ must be negative to be a "sink" rather than a "source" of flux.

(d) Prove that the optical theorem is valid when when inelastic events are included in the total cross section σ_t.