Rodeo algorithm for quantum computation

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Outline

Introduction

Single-qubit gates

Two-qubit gates

Adiabatic evolution

Rodeo algorithm

Two-state rodeo algorithm

Summary

Introduction

The basic element in quantum computation is the qubit, which is a simply a two-level quantum system.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

There are also extensions to systems with more than two levels, known as qudits. But we will focus on qubits in these lectures.

In general, our qubit will be in a general superposition of the two states.

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad |\psi\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

With proper normalization we have

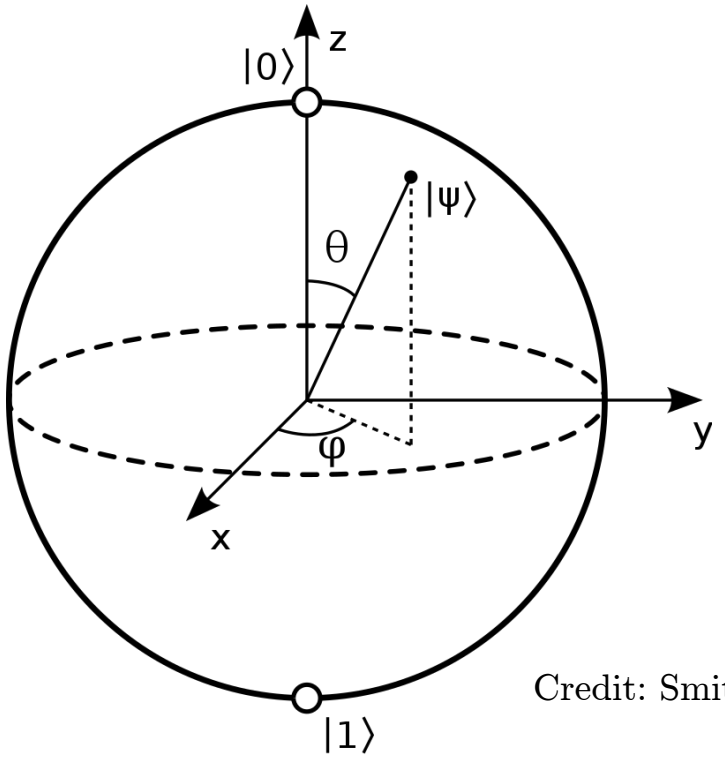
$$|\alpha|^2 + |\beta|^2 = 1$$

Up to an overall complex phase, we can write

$$|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\varphi}\sin(\theta/2)\,|1\rangle$$

$$0 \leq \theta \leq \pi,\ 0 \leq \varphi < 2\pi$$

This can be represented as a point on the Bloch sphere



Credit: Smite-Meister

For a two-qubit system we have the four basis states

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Any arbitrary state can be written as

$$|\psi\rangle = \alpha_{00}\,|00\rangle + \alpha_{01}\,|01\rangle + \alpha_{10}\,|10\rangle + \alpha_{11}\,|11\rangle$$

with normalization

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

For the N-qubit system, any arbitrary state can be written as

$$|\psi\rangle = \sum_{i_1 \in \{0,1\}} \cdots \sum_{i_N \in \{0,1\}} \alpha_{i_1 \cdots i_N} \, |i_1 \cdots i_N\rangle$$

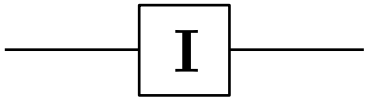
with normalization

$$\sum_{i_1 \in \{0,1\}} \cdots \sum_{i_N \in \{0,1\}} |\alpha_{i_1 \cdots i_N}|^2 = 1$$

One-qubit gates

Since the evolution of quantum systems is unitary, all quantum gates are unitary.

Identity gate



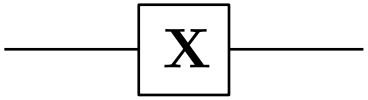
$$\mathbf{I}\,|0\rangle = |0\rangle\,,\ \mathbf{I}\,|1\rangle = |1\rangle$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\mathbf{I}^\dagger = \mathbf{I} = \mathbf{I}^{-1}$$

NOT gate (= Pauli-X gate)



If we view 0 and 1 as logical false and true, then the NOT gate corresponds to a logical negation or bit flip that exchanges 0 and 1.

$$\mathbf{X}\,|0\rangle = |1\rangle\,,\ \mathbf{X}\,|1\rangle = |0\rangle$$

$$\mathbf{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

$$\mathbf{X}^\dagger = \mathbf{X} = \mathbf{X}^{-1}$$

The $\mathbf{X}$ notation for NOT has a double meaning, since $\mathbf{X}$ can also be viewed as the Pauli-X gate.

Pauli-Y gate

$$\mathbf{Y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -i\beta \\ i\alpha \end{bmatrix}$$

$$\mathbf{Y}^\dagger = \mathbf{Y} = \mathbf{Y}^{-1}$$

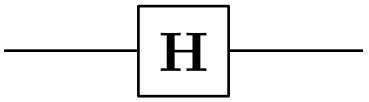
Pauli-Z gate

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$$

$$\mathbf{Z}^\dagger = \mathbf{Z} = \mathbf{Z}^{-1}$$

Hadamard gate



$$\mathbf{H}\left|0\right\rangle = \frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle,\ \mathbf{H}\left|1\right\rangle = \frac{1}{\sqrt{2}}\left|0\right\rangle - \frac{1}{\sqrt{2}}\left|1\right\rangle$$

$$\mathbf{H} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\mathbf{H}^{\dagger} = \mathbf{H} = \mathbf{H}^{-1}$$

Phase gate

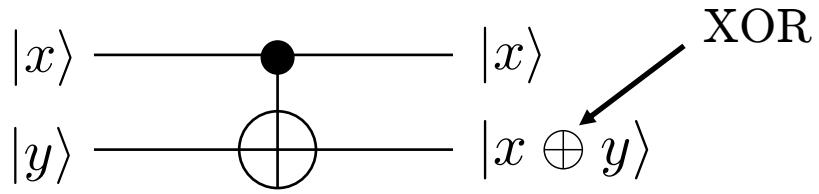
$$\mathbf{R}_\phi\left|0\right\rangle = \left|0\right\rangle,\ \mathbf{R}_\phi\left|1\right\rangle = e^{i\phi}\left|1\right\rangle$$

$$\mathbf{R}_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$$

$$\mathbf{R}_\phi^{\dagger} = \mathbf{R}_{-\phi} = \mathbf{R}_\phi^{-\mathbf{1}}$$

Two-qubit gates

Controlled-NOT (C-NOT) gate

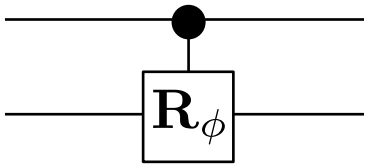


$$|00\rangle \to |00\rangle \qquad |01\rangle \to |01\rangle \qquad |10\rangle \to |11\rangle \qquad |11\rangle \to |10\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Controlled Phase gate

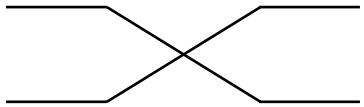


$$|00\rangle \to |00\rangle \qquad |01\rangle \to |01\rangle \qquad |10\rangle \to |10\rangle \qquad |11\rangle \to e^{i\phi}\,|11\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{bmatrix}$$

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

SWAP gate



$$|00\rangle \to |00\rangle \qquad |01\rangle \to |10\rangle \qquad |10\rangle \to |01\rangle \qquad |11\rangle \to |11\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Adiabatic evolution

The final Hamiltonian will be the Hamiltonian of interest. Choose an initial Hamiltonian with a simple ground state that can be easily prepared. We interpolate between these two Hamiltonians.

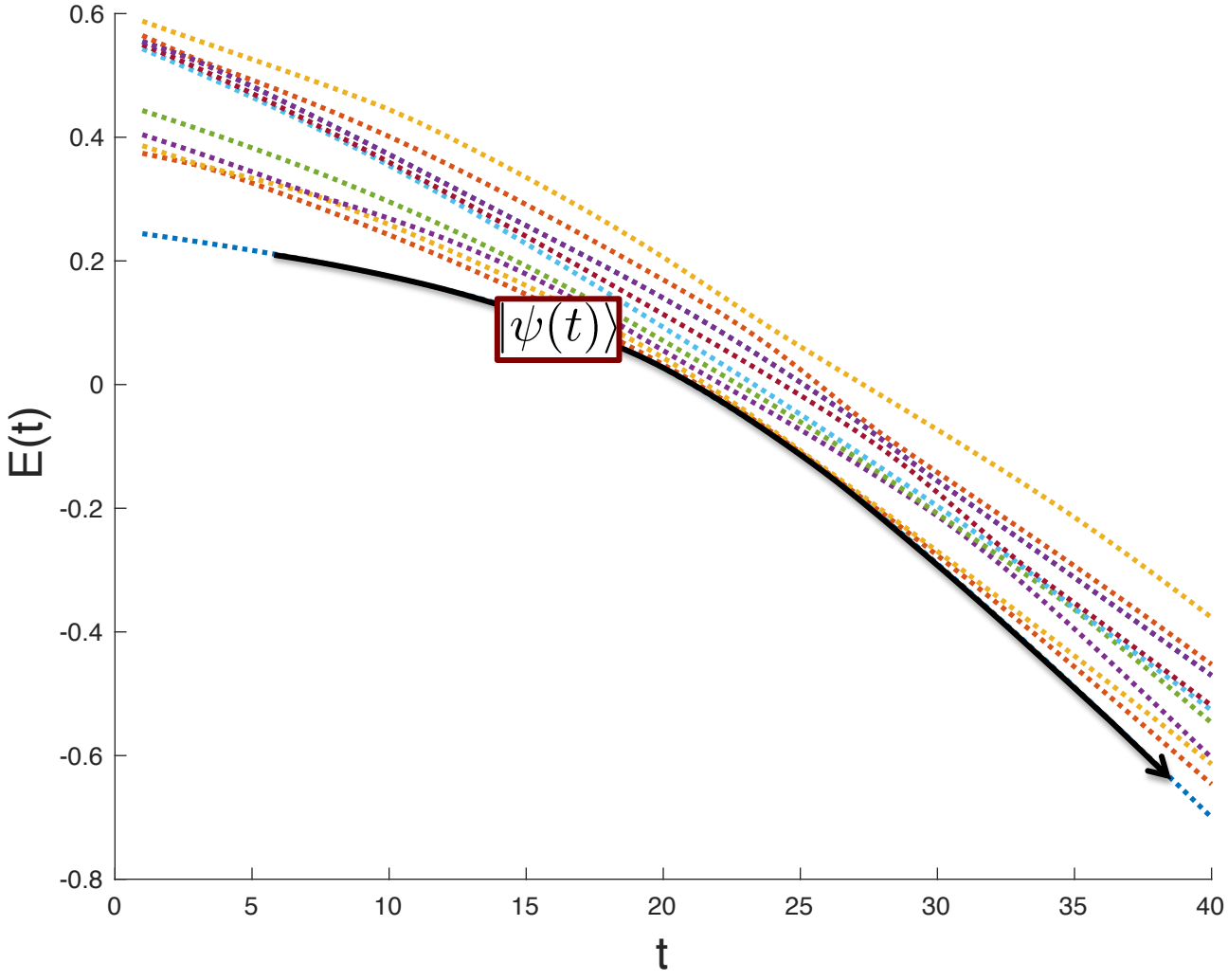
$$H(t=0) = H_I \qquad \leftarrow H(t) \rightarrow \qquad H(t=t_F) = H_F$$

In the limit of slow time evolution, we remain in an eigenstate of $H(t)$ throughout

$$H(t)\,|\psi(t)\rangle = E(t)\,|\psi(t)\rangle$$

At the end of the time evolution, we have

$$H_F\,|\psi(t_F)\rangle = E_F\,|\psi(t_F)\rangle$$

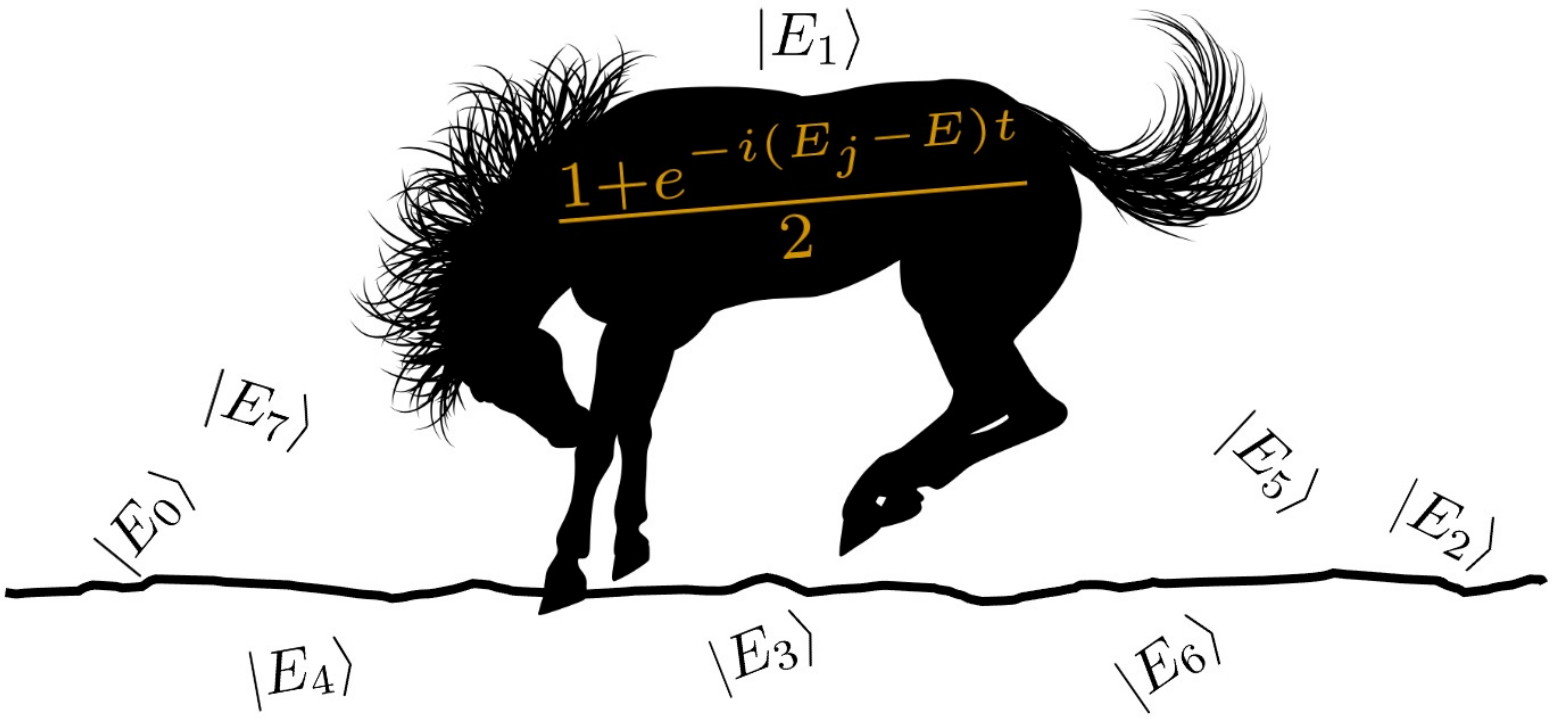

0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
E(t)
0
5
10
15
20
25
30
35
40
t
$|\psi(t)\rangle$

Unfortunately, adiabatic evolution is only practical for small systems with a substantial energy gap. The error in the wave function scales as

$$\|\Delta\psi\| \leq \max_{t\in[0,t_F]} \frac{\|\frac{d}{dt}H(t)\|}{|E_1(t)-E_0(t)|^2}$$

Farhi, Goldstone, Gutmann, and Sipser, quant-ph/0001106 (2000)
Roland and Cerf, Phys. Rev. A 65, 042308 (2002)
Wiebe and Babcock, New. J. Phys. 14, 013024, (2012)

Rodeo algorithm



Kenneth Choi, D.L., Joey Bonitati, Zhengrong Qian, Jacob Watkins, PRL 127, 040505 (2021)

Consider a single qubit and a unitary operation called the Hadamard gate

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = U^{\dagger} = U^{-1}$$

Consider another unitary operation that is a diagonal phase rotation

$$R(E_{\text{obj}}, E, t) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-it(E_{\text{obj}} - E)} \end{bmatrix}$$

We then have

$$U^{\dagger} R(E_{\text{obj}}, E, t) U = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} e^{-it(E_{\text{obj}} - E)} & \frac{1}{2} - \frac{1}{2} e^{-it(E_{\text{obj}} - E)} \\ \frac{1}{2} - \frac{1}{2} e^{-it(E_{\text{obj}} - E)} & \frac{1}{2} + \frac{1}{2} e^{-it(E_{\text{obj}} - E)} \end{bmatrix}$$

Let us now start in the $\begin{bmatrix}0\\1\end{bmatrix}$ state and perform these unitary operations

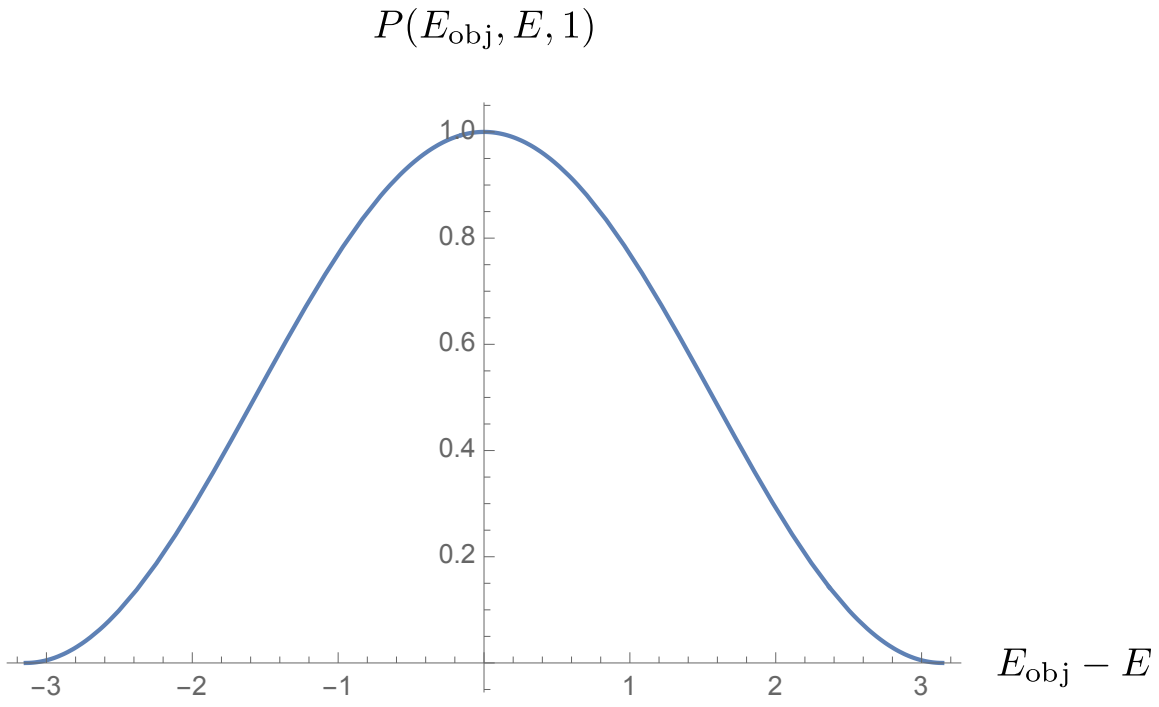
$$U^\dagger R(E_{\text{obj}}, E, t) U \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{1}{2} e^{-it(E_{\text{obj}} - E)} \\ \frac{1}{2} + \frac{1}{2} e^{-it(E_{\text{obj}} - E)} \end{bmatrix}$$

and then project back to the $\begin{bmatrix}0\\1\end{bmatrix}$ state

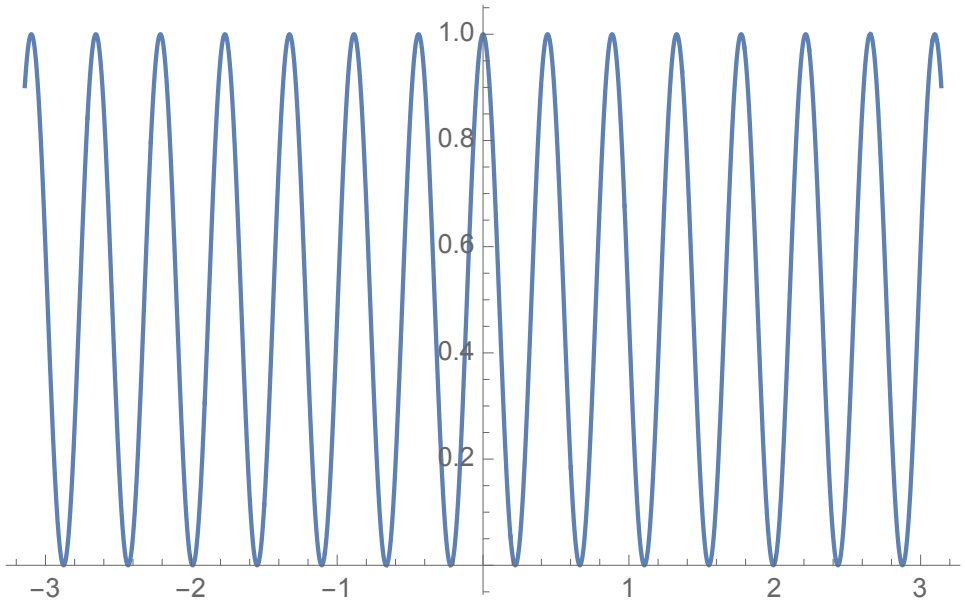
$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} U^\dagger R(E_{\text{obj}}, E, t) U \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} + \frac{1}{2} e^{-it(E_{\text{obj}} - E)} \end{bmatrix}$$

This projection is done via quantum measurement and the success probability is

$$P(E_{\text{obj}}, E, t) = \left| \frac{1}{2} + \frac{1}{2} e^{-it(E_{\text{obj}} - E)} \right|^2 = \cos^2 \left[\frac{t(E_{\text{obj}} - E)}{2} \right]$$

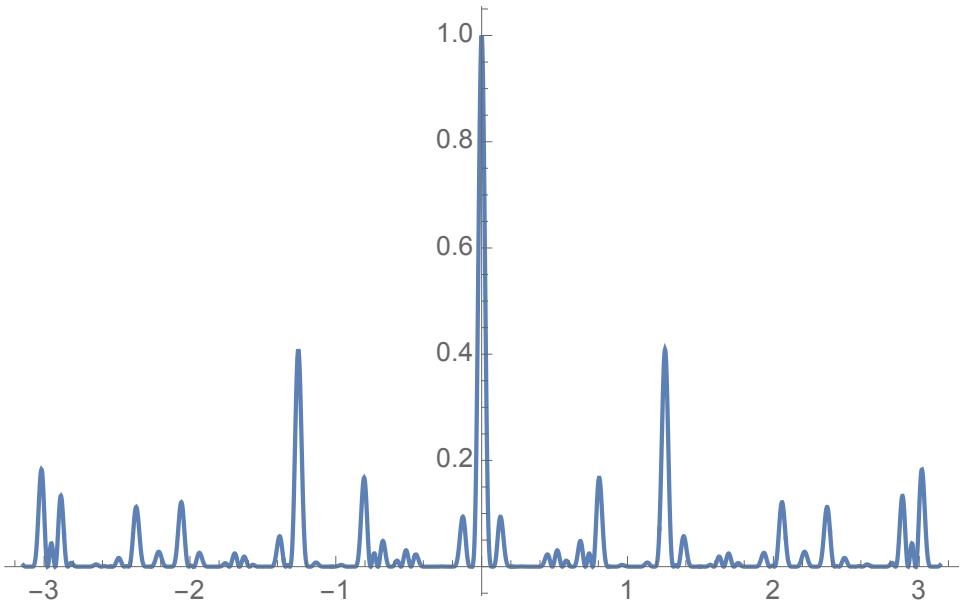

$P(E_{\mathrm{obj}}, E, 1)$
1.0
0.8
0.6
0.4
0.2
−3
−2
−1
1
2
3
$E_{\mathrm{obj}} - E$

$P(E_{\mathrm{obj}}, E, 14.2023)$

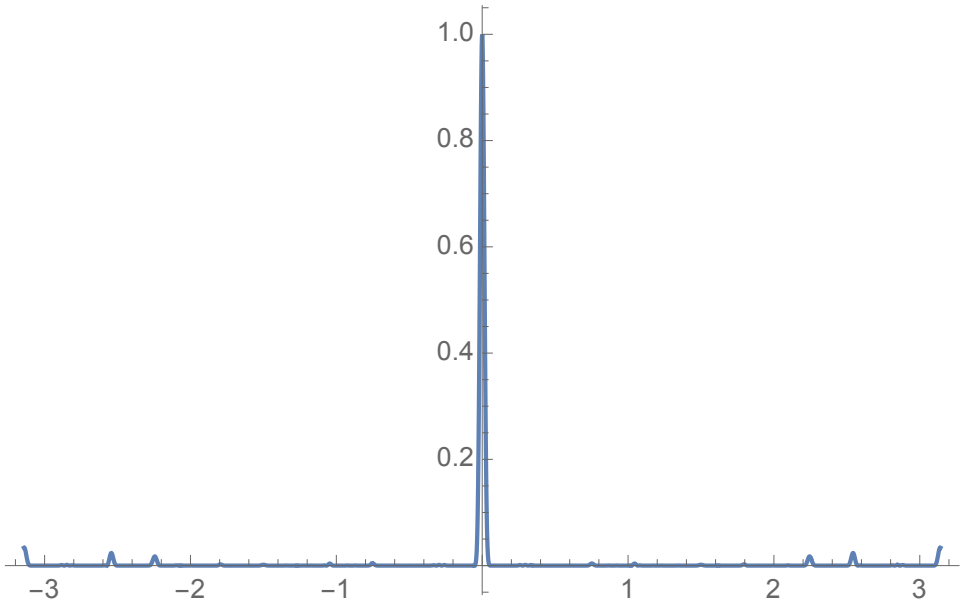


$E_{\mathrm{obj}} - E$

$$\prod_{k=1}^{5} P(E_{\mathrm{obj}}, E, t_k) \qquad |t_k| < 50$$



$$\prod_{k=1}^{10} P(E_{\mathrm{obj}}, E, t_k) \qquad |t_k| < 50$$



Let us couple this qubit, which we call the "arena" or "ancilla" qubit, to another system that we call the "object". We also promote the 2 x 2 matrices to become 2 x 2 matrices of operators acting on the object.

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{-it(E_{\mathrm{obj}}-E)} \end{bmatrix} \rightarrow \begin{bmatrix} \hat{I} & 0 \\ 0 & e^{-it(\hat{H}_{\mathrm{obj}}-E)} \end{bmatrix}$$

We then consider the same combination

$$\begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \hat{I} & 0 \\ 0 & e^{-it(\hat{H}_{\mathrm{obj}}-E)} \end{bmatrix} \begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix}$$

We start from the state $\begin{bmatrix} 0 \\ |\psi_{\rm init}\rangle \end{bmatrix}$ and we perform the operations and then measure if the arena qubit is in the $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ state

$$\begin{bmatrix} 0 & 0 \\ 0 & \hat{I} \end{bmatrix} \begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \hat{I} & 0 \\ 0 & e^{-it(\hat{H}_{\rm obj}-E)} \end{bmatrix} \begin{bmatrix} \frac{\hat{I}}{\sqrt{2}} & \frac{\hat{I}}{\sqrt{2}} \\ \frac{\hat{I}}{\sqrt{2}} & -\frac{\hat{I}}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ |\psi_{\rm init}\rangle \end{bmatrix} = \begin{bmatrix} 0 \\ [\frac{1}{2} + \frac{1}{2}e^{-it(\hat{H}_{\rm obj}-E)}]\,|\psi_{\rm init}\rangle \end{bmatrix}$$

By repeated successful measurements with random values of t, we reduce the spectral weight of eigenvectors with energies that do not match E.

The convergence is exponential. For N cycles of the rodeo algorithm, the suppression factor for undesired energy states is $1/4^N$.

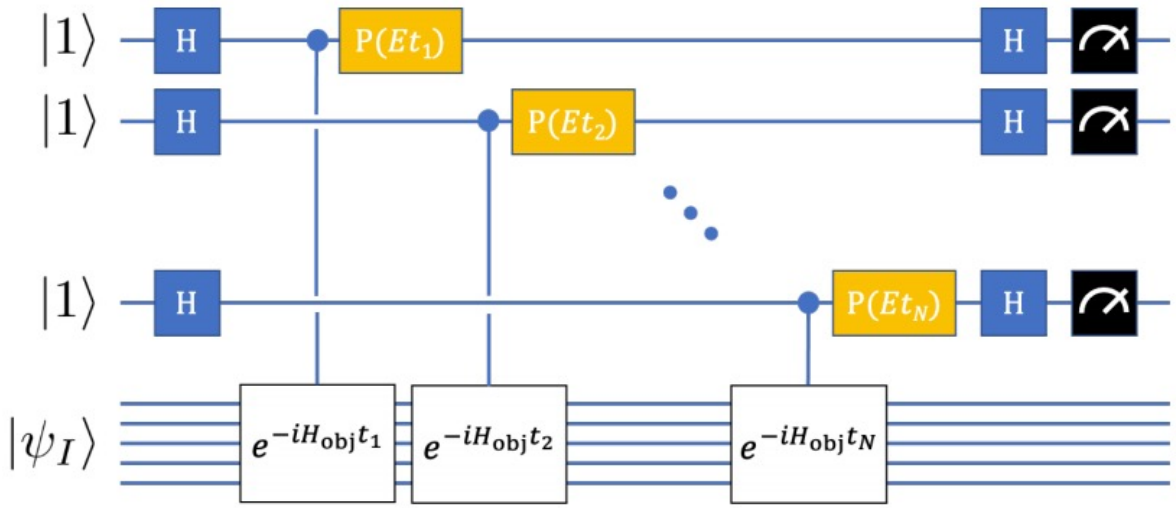


FIG. 1. (color online) **Circuit diagram for the rodeo algorithm.** The object system starts in an arbitrary state $|\psi_I\rangle$. Each of the arena qubits are initialized in the state $|1\rangle$ and operated on by a Hadamard gate H. We use each arena qubit $n = 1, \cdots, N$ for the controlled time evolution of the object Hamiltonian, $H_{\rm obj}$, for time t_n. This is followed by a phase rotation $\mathrm{P}(Et_n)$ on arena qubit n, another Hadamard gate H, and then measurement.

Kenneth Choi, D.L., Joey Bonitati, Zhengrong Qian, Jacob Watkins, PRL 127, 040505 (2021)

Initial-state spectral function and state preparation. The example shown below is for a 1D Heisenberg chain with ten sites, antiferromagnetic interactions, and uniform magnetic field.

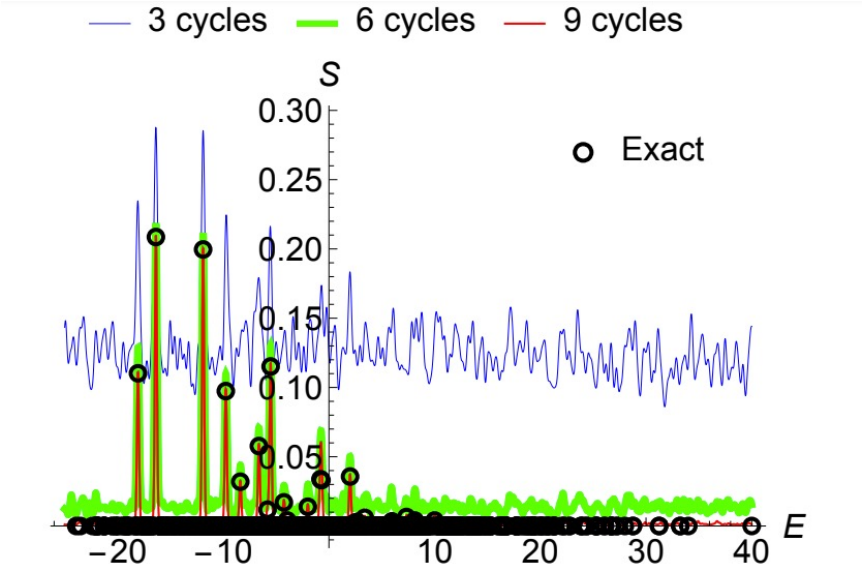


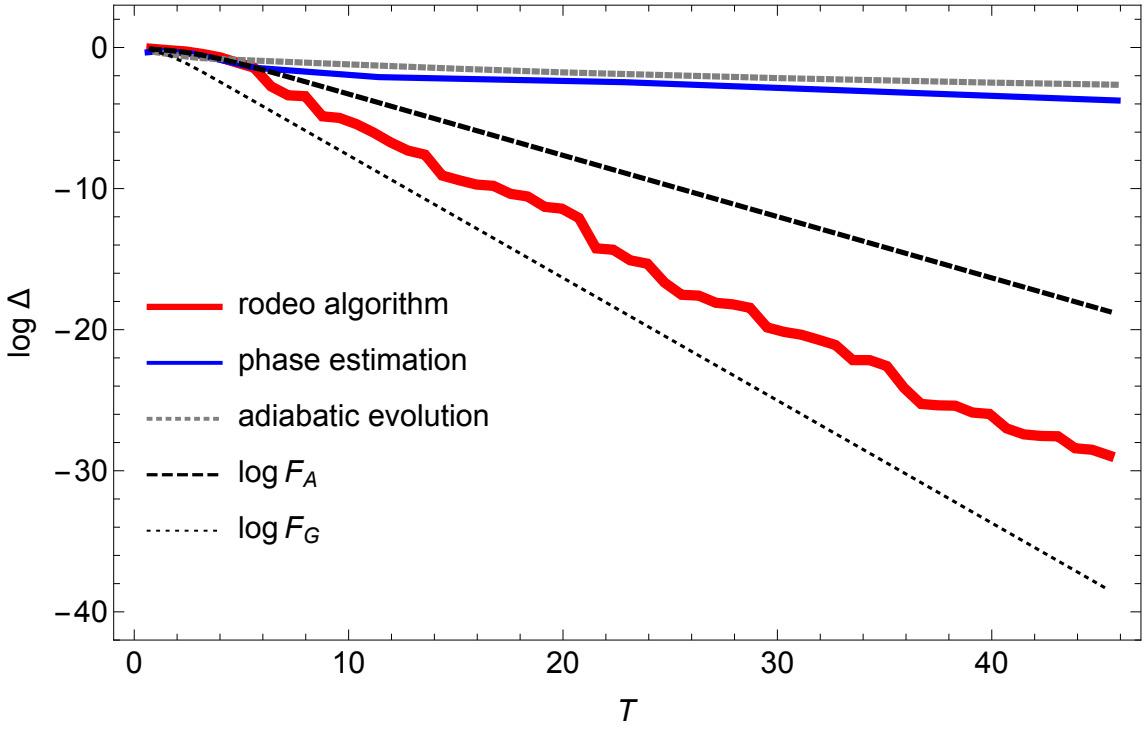
FIG. 4. (color online) **Initial-state spectral function for the Heisenberg model.** We plot the initial-state spectral function using the rodeo algorithm for the Heisenberg spin chain with 3 (thin blue line), 6 (thick green line), and 9 (medium red line) cycles. We have averaged over 20 sets of Gaussian random values for t_n with $t_{\mathrm{RMS}} = 5$. For comparison, we also show the exact initial-state spectral function with black open circles.

$$|\psi_{\mathrm{init}}\rangle = |0101010101\rangle$$

TABLE I. Overlap probability with energy eigenvector $|E_j\rangle$ after N cycles of the rodeo algorithm using Gaussian random values for t_n with $t_{\mathrm{RMS}} = 5$ and $E = E_j$.

E_j	$N = 0$	$N = 3$	$N = 6$	$N = 9$
−18.1	0.110	0.746	0.939	0.997
−16.4	0.209	0.841	0.993	1.000
−11.9	0.200	0.629	0.889	0.999
−9.76	0.0974	0.488	0.903	0.999
−8.38	0.0320	0.467	0.832	0.993
−6.63	0.0577	0.309	0.818	0.996
−5.81	0.0118	0.179	0.637	0.817
−5.52	0.115	0.456	0.766	0.997
−4.26	0.0171	0.144	0.696	0.995
−3.95	0.00401	0.0430	0.343	0.952
−2.00	0.0139	0.158	0.593	0.942
−0.802	0.0338	0.216	0.545	0.594
−0.704	0.0331	0.286	0.540	0.585
2.00	0.0357	0.371	0.925	0.994
2.42	0.00235	0.0122	0.0874	0.521
2.68	0.00291	0.0845	0.639	0.929
3.39	0.00592	0.0360	0.754	0.943
5.96	0.00336	0.0951	0.559	0.981
7.33	0.00650	0.184	0.792	0.978
8.13	0.00393	0.0832	0.665	0.841
8.24	0.00105	0.0275	0.142	0.289
10.0	0.00397	0.0128	0.295	0.902

Comparison with other well-known algorithms. Let Δ be the norm of the error in the wave function.



$$F_A \equiv \sqrt{2^{-N}(1-p)/[p+2^{-N}(1-p)]}$$

$$F_G \equiv \sqrt{4^{-N}(1-p)/[p+4^{-N}(1-p)]}$$

Preconditioning with adiabatic evolution

The computational effort needed for the rodeo algorithm is inversely proportional to the overlap probability between the initial state and the desired eigenvector.

We can use adiabatic evolution to increase this overlap probability.

TABLE I. Overlap probability with energy eigenvector $|E_j\rangle$ with $E = E_j = -18.1$ after preconditioning with adiabatic evolution for time t_{AE} and the applying N cycles of the rodeo algorithm using Gaussian random values for t_n with $t_{\mathrm{rms}} = 5$.

E_j	t_{AE}	$N = 0$	$N = 3$	$N = 6$	$N = 9$
−18.1	0	0.110	0.746	0.939	0.997
−18.1	5	0.83074	0.99875	0.99988	0.99999

Choi, D.L., Bonitati, Qian, Watkins, Phys. Rev. Lett. 127, 040505 (2021)

Demonstration on a quantum computer

Demonstration of the Rodeo Algorithm on a Quantum Computer

Zhengrong Qian,[1] Jacob Watkins,[1] Gabriel Given,[1] Joey Bonitati,[1] Kenneth Choi,[2] and Dean Lee[1]

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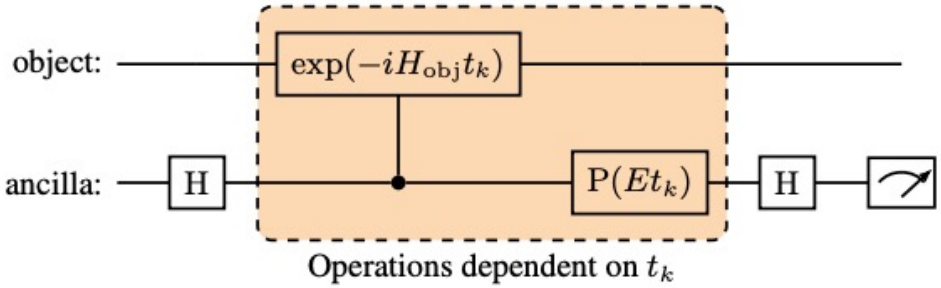
The rodeo algorithm is an efficient algorithm for eigenstate preparation and eigenvalue estimation for any observable on a quantum computer. The only requirement is that the initial state has sufficient overlap probability with the desired eigenstate. While it is exponentially faster than well-known algorithms such as phase estimation and adiabatic evolution for eigenstate preparation, it has yet to be implemented on an actual quantum device. In this work, we apply the rodeo algorithm to determine the energy levels of a random one-qubit Hamiltonian, resulting in a relative error of 0.08% using mid-circuit measurements on the IBM Q device Casablanca. This surpasses the accuracy of directly-prepared eigenvector expectation values using the same quantum device. We take advantage of the high-accuracy energy determination and use the Hellmann-Feynman theorem to compute eigenvector expectation values for different random one-qubit observable. For the Hellmann-Feynman calculations, we find a relative error of 0.7%. We conclude by discussing possible future applications of the rodeo algorithm for multi-qubit Hamiltonians.

arXiv: 2110.07747

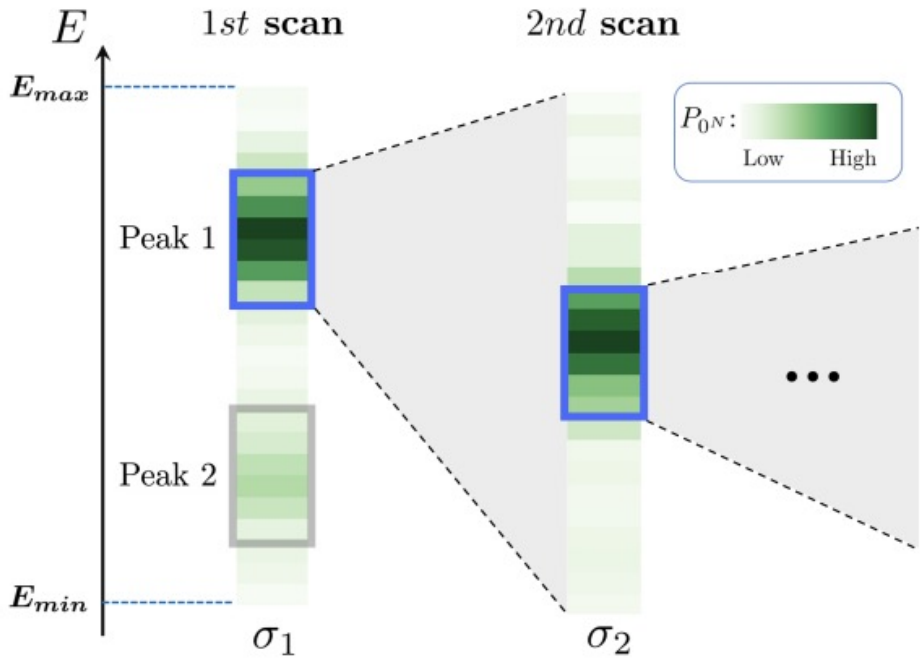
Using IBM Q devices, we implement the rodeo algorithm for a one qubit Hamiltonian. We consider a random Hamiltonian of the form

$$H_{\text{obj}} = H^{(0)} = -0.08496I - 0.89134X + 0.26536Y + 0.57205Z$$

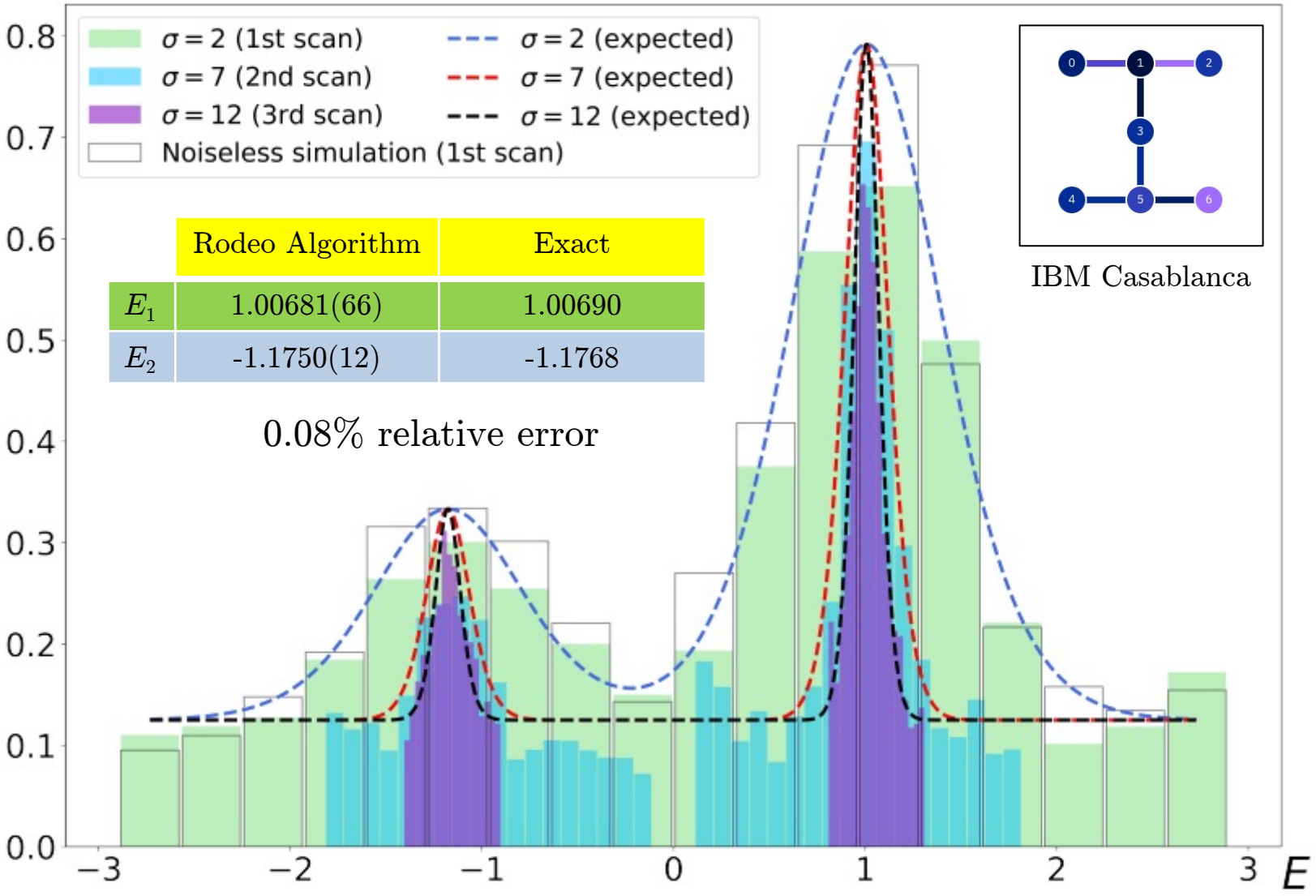
We use mid-circuit measurements without resets for the ancilla qubit



Each circuit consists of three cycles of the rodeo algorithm, corresponding to three controlled time evolutions and three ancilla qubit measurements. We sweep through the target energy E to perform an energy scan of the spectrum. We perform three separate scans of the energy, each time zooming in with more resolution.



Single qubit Hamiltonian



	Rodeo Algorithm	Exact
E_1	1.00681(66)	1.00690
E_2	-1.1750(12)	-1.1768

0.08% relative error

IBM Casablanca

Z. Qian, J. Watkins, G. Given, J. Bonitati, K. Choi, D.L., arXiv:2110.07747

We also use the Hellmann-Feynman theorem to compute eigenvector expectation values for a different one qubit observable.

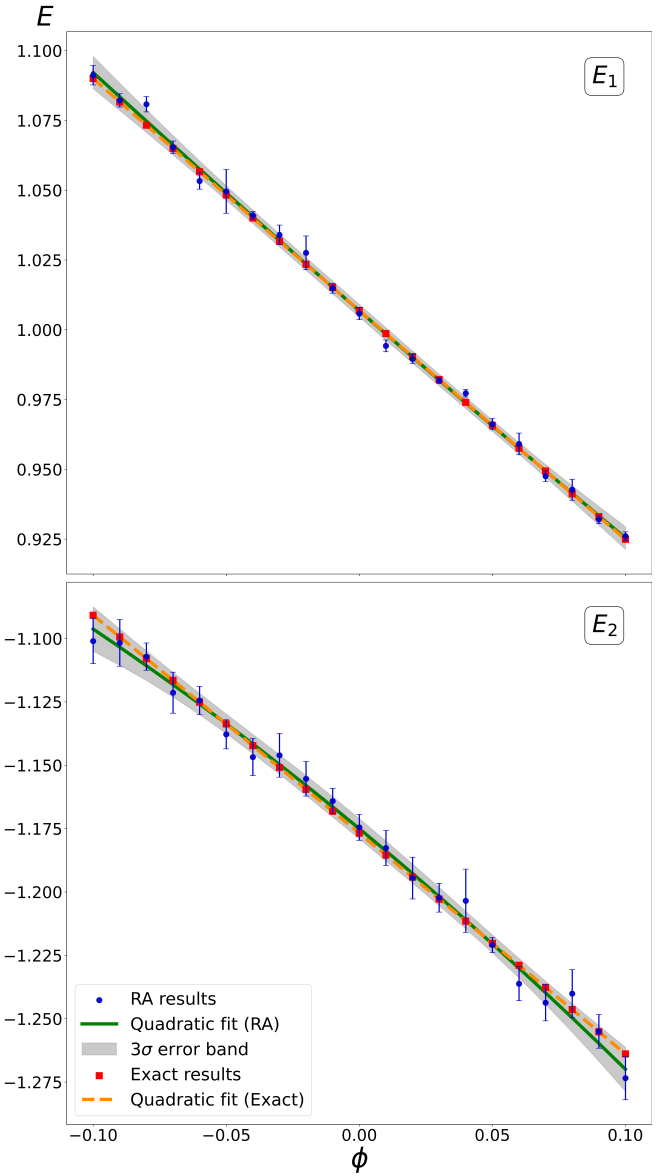
$$H_{\text{obj}}(\phi) = H^{(0)} + \phi H^{(1)}$$

$$H^{(0)} = -0.08496I - 0.89134X + 0.26536Y + 0.57205Z$$

$$H^{(1)} = -0.84537I + 0.00673X - 0.29354Y + 0.18477Z$$

$$H^{(0)} \left|\psi_n^{(0)}\right\rangle = E_n \left|\psi_n^{(0)}\right\rangle$$

$$\left.\frac{dE_n(\phi)}{d\phi}\right|_{\phi=0} = \left\langle\psi_n^{(0)}\right| H^{(1)} \left|\psi_n^{(0)}\right\rangle$$



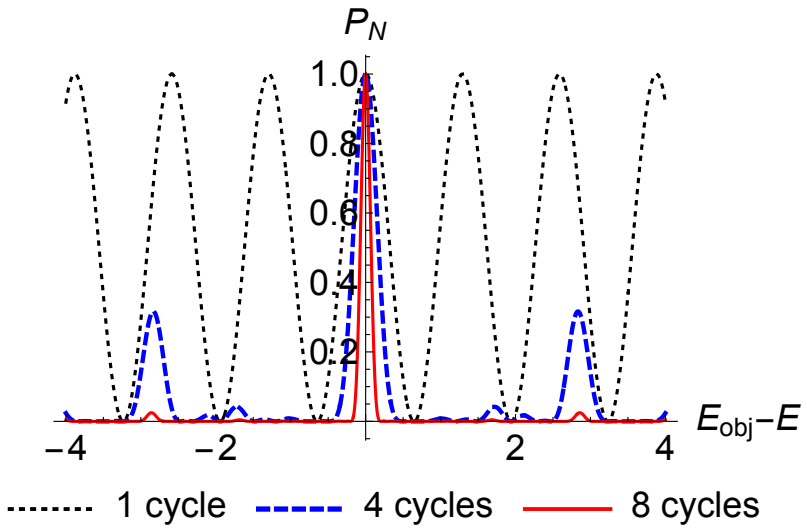
	$\lvert\psi_1(0)\rangle$	exact	$\lvert\psi_2(0)\rangle$	exact
$\langle H^{(0)}\rangle$	1.00681(66)	1.00690	−1.1750(12)	−1.1768
$\langle H^{(1)}\rangle$	−0.8338(89)	−0.8254	−0.868(14)	−0.8653

0.7% relative error

Two-state rodeo algorithm

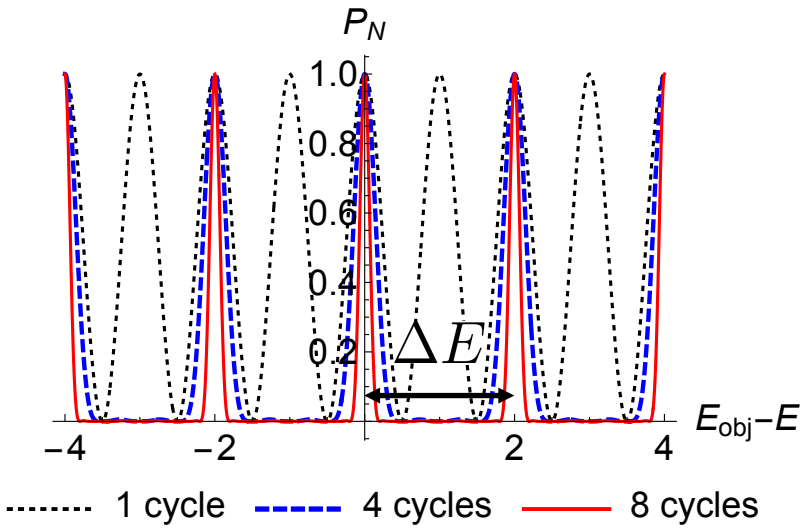
$$\prod_k P(E_{\rm obj}, E, t_k)$$

t_k unconstrained



rodeo algorithm

$t_k = \text{integer} \cdot 2\pi/\Delta E$



two-state rodeo algorithm

Bee-Lindgren, et al., work in progress

After applying the two-state rodeo algorithm, the quantum state will be an approximate superposition of two energy eigenstates

$$|\psi\rangle \approx c_j \, |E_j\rangle + c_i \, |E_i\rangle$$

$$\Delta E = E_j - E_i$$

We define the phases of the energy eigenstates so that the inner products with the initial state is real and positive. We then have

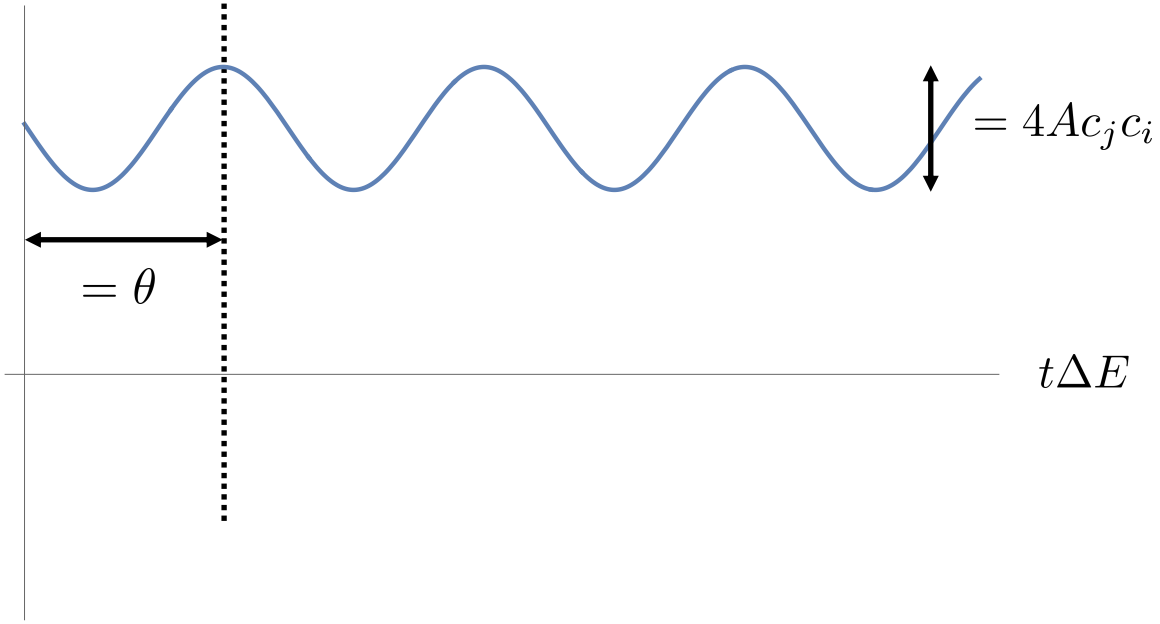
$$c_j > 0, \quad c_i > 0,$$

and their values are given by the square root of the corresponding probability peak in the initial-state spectral function.

Suppose we now evolve this state in time and compute the expectation value of some Hermitian operator O as a function of time. We will find an oscillatory signal from which we can determine the amplitude and phase of the transition matrix element.

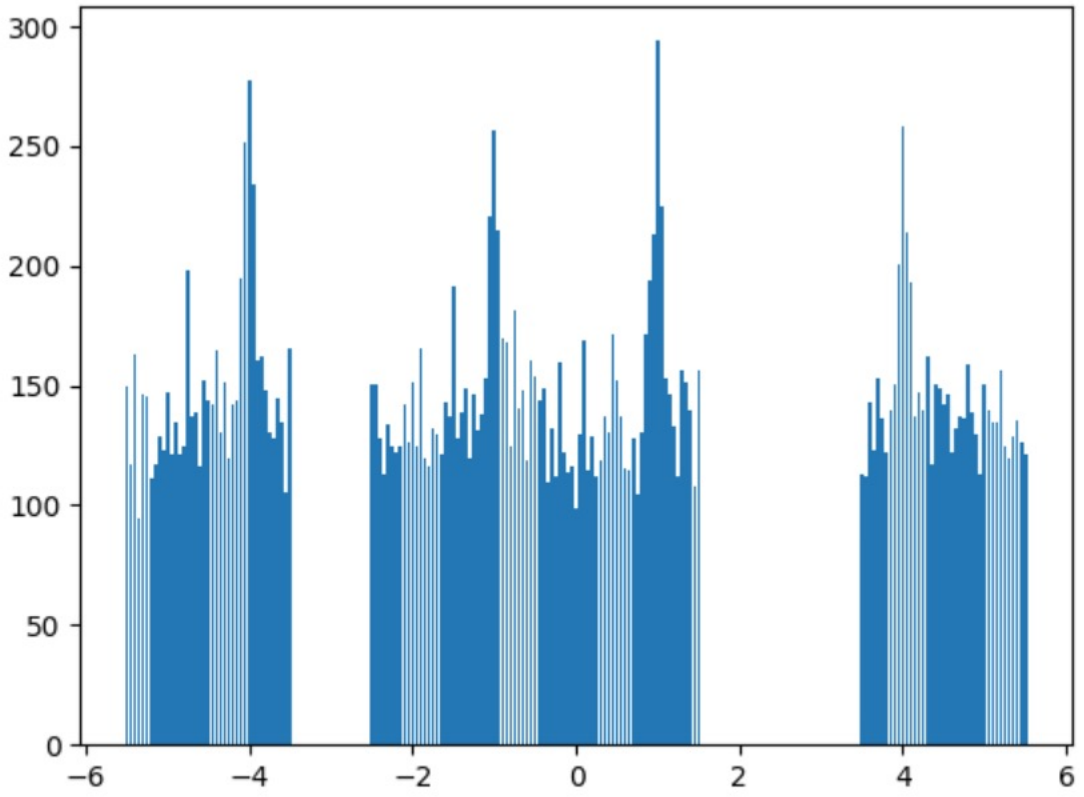
$$\langle E_j|O|E_i\rangle = Ae^{i\theta}$$

$\langle\psi(t)|O|\psi(t)\rangle$



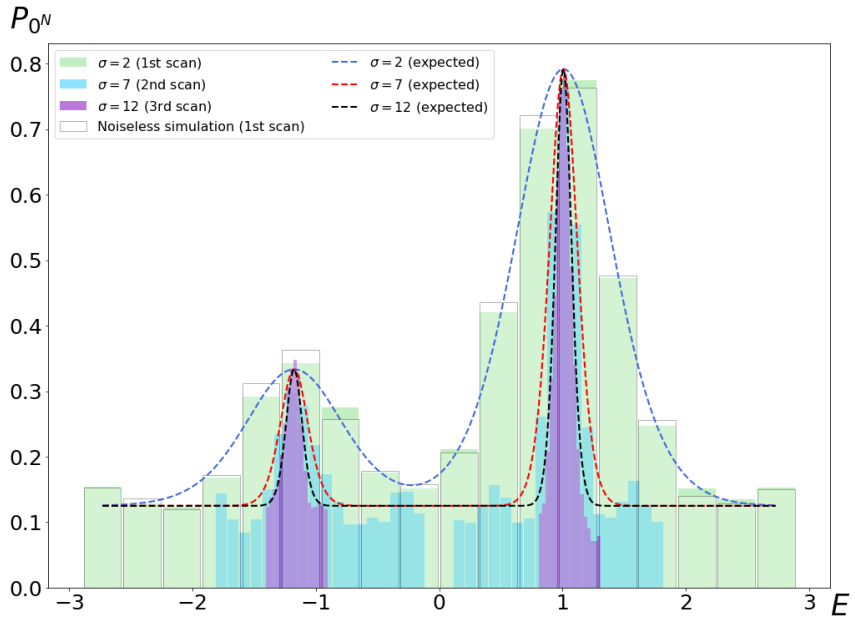
Implementation on larger systems

Two qubit Hamiltonian using IBM Q:

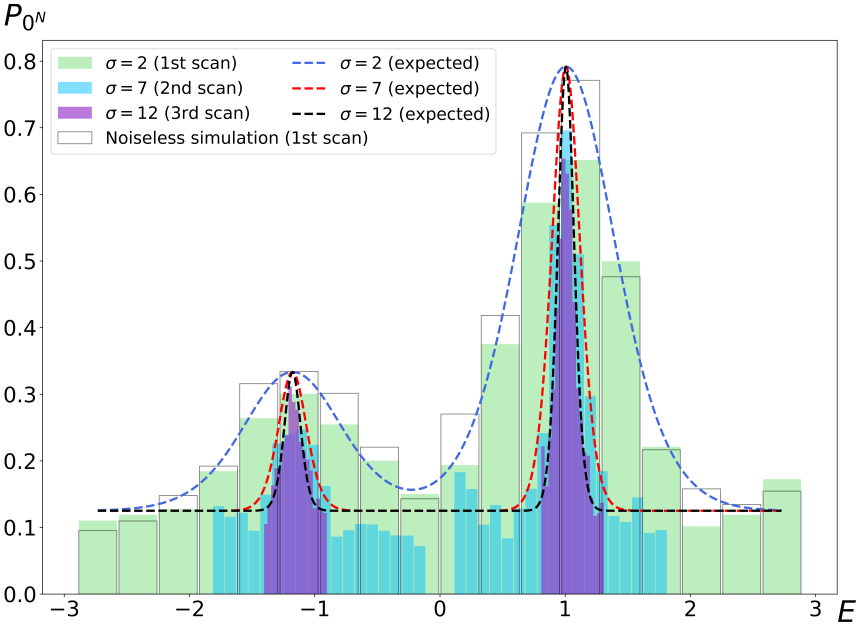


Bee-Lindgren, et al., work in progress

Results obtained by Quantinuum theory group on Honeywell H1 system:



Results on IBM Q Casablanca:



Summary

We started by reviewing the basics of quantum gates. We discussed adiabatic theorem and adiabatic evolution. We then presented a method called the rodeo algorithm that can prepare any energy eigenstate on a quantum computer and determine the full energy spectrum.

The method is exponentially faster than other well-known algorithms for quantum state preparation. It requires only one ancilla qubit to perform controlled time evolutions and is resilient against noise. We have discussed several applications and tests that demonstrate the actual performance.