JÜLICH
FORSCHUNGSZENTRUM

Jacobi no-core Shell Model (J-NCSM)

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Ab initio nuclear structure calculations

Goal: solving the non-relativistic A-body Schrödinger Eq. for bound states

$$\mathrm{H}\,|\Psi_n\rangle = \mathrm{E}_n\,|\Psi_n\rangle, \qquad \mathrm{H} = \sum_{i=1}^{A} \frac{\mathbf{k}_i^2}{2m} + \sum_{i<j=1}^{A} \mathrm{V}_{ij} + \sum_{i<j<k=1}^{A} \mathrm{V}_{ijk}$$

ab initio:

- protons and neutrons are fundamental degrees of freedom
- realistic (microscopic) nuclear interactions as in put
- controlled and improvable truncations

ab initio (few)many-body approaches:

- Faddeev-Yakubovsky integral equation, exact solutions for $A \leq 4$, arXiv:nucl-th/0004023
- Green's function Monte Carlo, require local interactions, arXiv:0804.3501
- No-core shell model (NCSM), arXiv:0904.0463
- Coupled-cluster method, arXiv:1312.7872

→ need large model space (soft interactions)

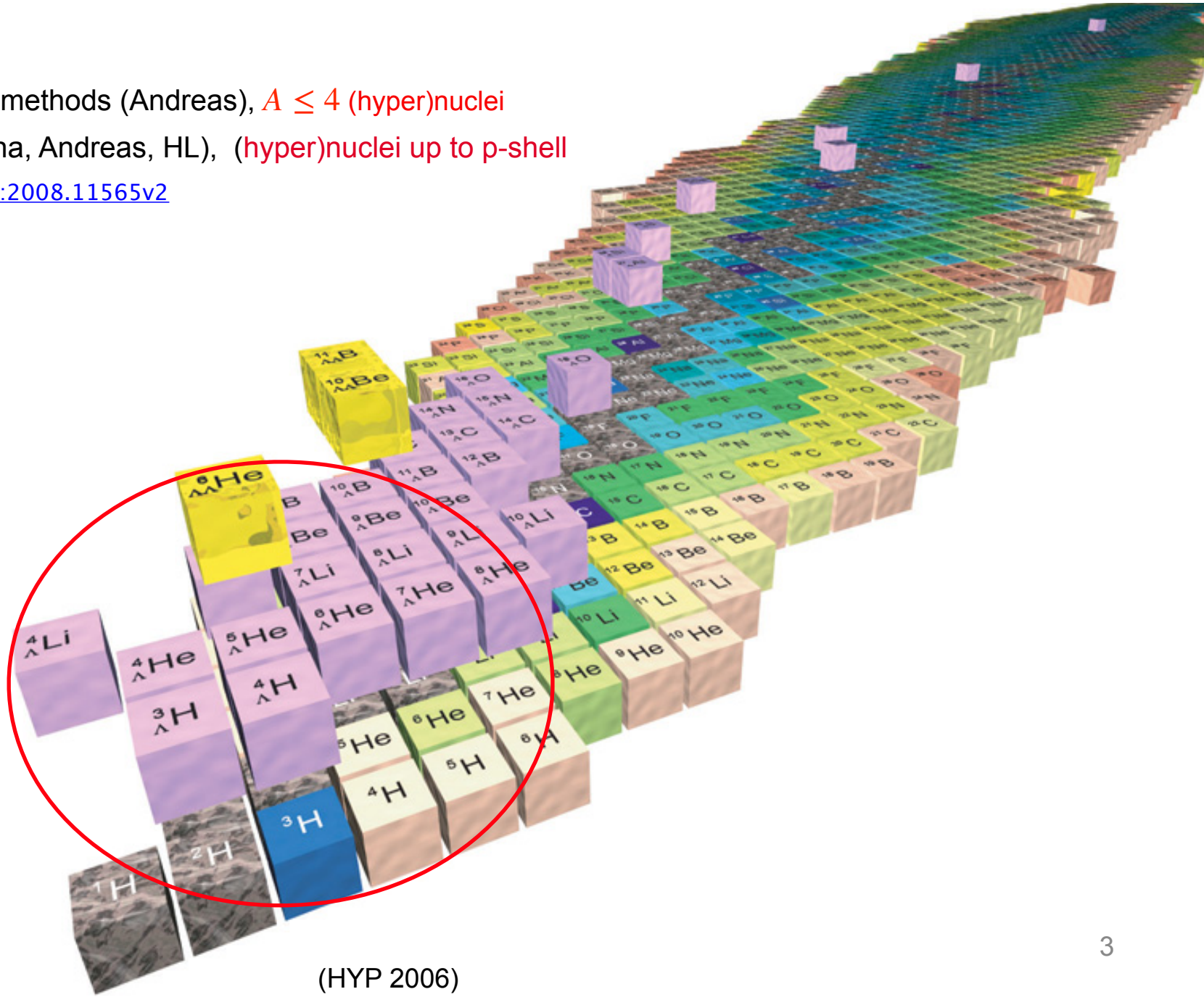
- Lattice Monte Carlo, suitable for states with complex geometries, arXiv:0804.3501
- …

Ab initio nuclear structure calculations

at our disposal:

- Faddeev-Yakubovsky methods (Andreas), $A \leq 4$ (hyper)nuclei
- Jacobi NCSM (Susanna, Andreas, HL), (hyper)nuclei up to p-shell

arXiv:1510.06070v1 , arXiv:2008.11565v2



(HYP 2006)

No-core shell model (NCSM)

- Idea: represent the A-body translationally invariant Hamiltonian

$$H_{int} = \sum_{i=1}^{A} \frac{\mathbf{k}_i^2}{2m} + \sum_{i<j=1}^{A} V_{ij} + \sum_{i<j<k=1}^{A} V_{ijk} - \frac{\mathbf{P}^2}{2M}$$
$$= \sum_{i<j=1}^{A} \frac{2}{A}\frac{\mathbf{p}_{ij}^2}{m} + \sum_{i<j=1}^{A} V_{ij} + \sum_{i<j<k=1}^{A} V_{ijk}, \qquad \mathbf{p}_{ij} = \frac{1}{2}(\mathbf{k}_i - \mathbf{k}_j)$$

in a basis constructed from HO functions, $\phi_{nlm}(\mathbf{p}) = \langle \mathbf{p} \,|\, nlm \rangle = R_{nl}(p) Y_{lm}(\hat{p})$

$$R_{nl}(p) = (-1)^n \sqrt{\frac{2(n!)b^3}{\Gamma(n+l+2/3)}} (pb)^l e^{-(pb)^2/2} L_n^{l+1/2}((pb)^2), \qquad b = 1/\sqrt{\mu\omega}$$

HO basis →
- essential for evaluating Hamiltonian matrix elements ✓
  (facilitate transformations between different bases)
- wrong asymptotic behavior, NCSM results converge slowly ⇒ require soft interactions ✓

- two approaches to construct basis states:
  - Slater determinant basis depending on single-particle coordinates (m-scheme importance truncated NCSM)
    - → antisymmetric, but contain CM motion ⇒ large dimension
    - → importance truncated basis (IT-NCSM) for p-shell
  - Jacobi basis expressed in relative Jacobi coordinated (Jacobi NCSM) ✓
    - → preserve translational symmetry of H, no CM motion, ⇒ small dimension
    - → antisymmetrization of basis states is demanding $A \leq 8$
- All particles are active (no inert core) ⇒ directly employ realistic BB interactions
- H is diagonalised via Lanczos iterations. Converge to exact solutions from above. ✓

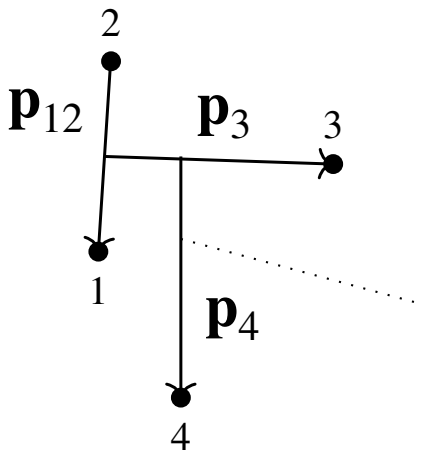
Jacobi NCSM basis

- Jacobi coordinates:

$$\mathbf{p}_{12} = \frac{m_2}{m_1+m_2}\mathbf{k}_1 - \frac{m_1}{m_1+m_2}\mathbf{k}_2,$$

$$\mathbf{p}_3 = \frac{m_1+m_2}{m_1+m_2+m_3}\mathbf{k}_3 - \frac{m_3}{m_1+m_2+m_3}(\mathbf{k}_1+\mathbf{k}_2)$$

$$\ldots$$



- Two-body Jacobi HO basis: $|\alpha_{12}\rangle$

$$|\alpha_{12}\rangle \equiv |N_{12}\,\underbrace{(l_{12}S_{12})J_{12}}_{JJ-coupling},(t_1t_2)t_{12}\rangle$$

antisymmetric → $(-1)^{l_{12}+s_{12}+t_{12}} = -1$

N_{12}: total HO energy quantum number (model space size): $N_{12} = 2n_{12} + l_{12}$

→ in partial-wave decomposed Jacobi-momentum basis (NN interactions are defined): $|p_{12}\,(l_{12}S_{12})J_{12}t_{12}\rangle$

$$|\alpha_{12}\rangle = \int dp_{12}p_{12}^2 R_{n_{12}l_{12}}(p_{12})\,|p_{12}(l_{12}S_{12})J_{12}\,t_{12}\,\rangle$$

$$\Rightarrow \langle\alpha_{12}|V_{NN}|\alpha'_{12}\rangle = \int dp_{12}dp'_{12}p_{12}^2\,p_{12}'^2\,R_{n_{12}l_{12}}(p_{12})R^*_{n'_{12}l'_{12}}(p'_{12})\,\langle p_{12}(l_{12}S_{12})J_{12}\,t_{12}|V_{NN}|p'_{12}(l'S_{12})J_{12}t'_{12}\rangle$$

dependence on HO ω

Jacobi NCSM basis

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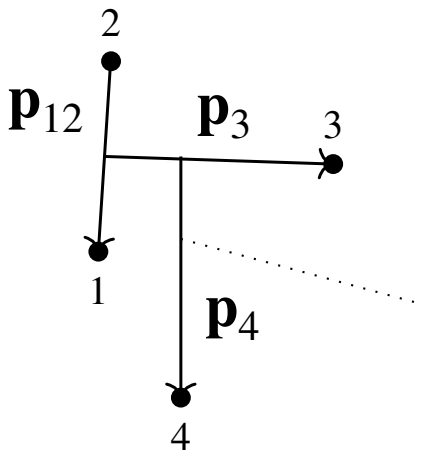
$$\ldots$$



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antisymmetric →

$$(-1)^{l_{12}+s_{12}+t_{12}} = -1$$

- Three-body Jacobi HO basis: $|\alpha_3 (N_3J_3T_3)\rangle$

$$|\alpha_3^{*(1)}\rangle \equiv |N_3J_3T_3\,\alpha_{12}\,n_3(l_3s_3)I_3, t_3;\ (J_{12}I_3)J_3\,, (t_{12}t_3)T_3\rangle$$

→ complete, only antisymmetric w.t. r. $(1) \leftrightarrow (2)$

$$N_3 = N_{12} + n_3 + 2l_3$$

Jacobi NCSM basis

- Jacobi coordinates:

$$\mathbf{p}_{12} = \frac{m_2}{m_1+m_2}\mathbf{k}_1 - \frac{m_1}{m_1+m_2}\mathbf{k}_2,$$

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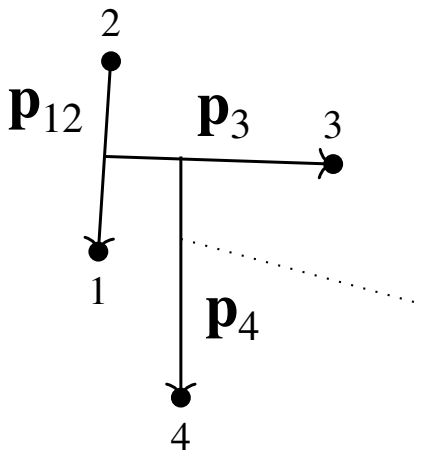

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→ complete, only antisymmetric w.t. r. $(1) \leftrightarrow (2)$

$$N_3 = N_{12} + n_3 + 2l_3$$

→ idea: use antisymmetrizer to project out the (non)antrisymmetric states

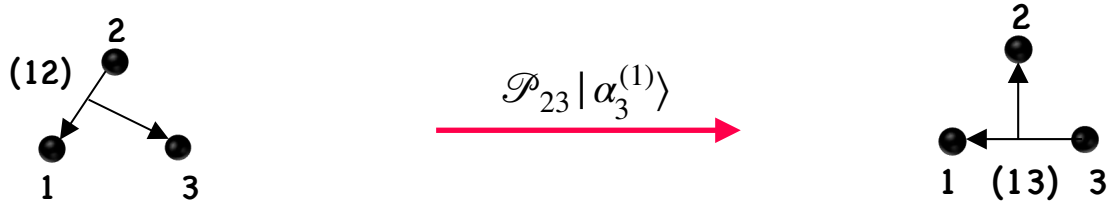
$$\langle\alpha_3'^{*(1)}|\frac{1}{3}(1-2\mathscr{P}_{23})|\alpha_3^{*(1)}\rangle\langle\alpha_3^{*(1)}|\alpha_3\rangle = \lambda\langle\alpha_3^{*(1)}|\alpha_3\rangle \qquad \lambda = (0)1$$

⇒ totally antisymmetrized 3-body basis $|\alpha_3\rangle \equiv |iN_3J_3T_3\rangle = \sum_{\alpha_3^{*(1)}} \langle\alpha_3^{*(1)}|\alpha_3\rangle|\alpha_3^{*(1)}\rangle, \qquad \lambda = 1$

coeff. of fractional parentage (cfp)
ω-independent

Jacobi NCSM basis

- Apply permutation operator $\mathscr{P}_{23}$ to $|\alpha_3^{*(1)}\rangle$



$$|\alpha_3^{*(1)}\rangle \equiv |\alpha_{12}\, n_3(l_3 s_3) I_3, t_3;\ (J_{12} I_3) J\,, (t_{12} t_3) T\rangle$$

$$|\beta_3^{*(1)}\rangle \equiv |\alpha_{13}\, n_2(l_2 s_2) I_2, t_2;\ (J_{13} I_2) J\,, (t_{13} t_2) T\rangle$$

$\Rightarrow$

$$\langle \alpha_3'^{*(1)} | P_{23}) | \alpha_3^{*(1)}\rangle = \langle \alpha_3'^{*(1)} | \beta_3^{*(1)}\rangle \quad \leftarrow \text{orthogonal basis transformation}$$

$$= \hat{J}_{13}\,\hat{I}_2\,\hat{J}_{12}\,\hat{I}_3 \sum_{LS} \hat{L}^2\,\hat{S}^2 \begin{Bmatrix} l_{13} & S_{13} & J_{13} \\ l_2 & s_2 & I_2 \\ L & S & J \end{Bmatrix} \begin{Bmatrix} l_{12} & S_{12} & J_{12} \\ l_3 & s_3 & I_3 \\ L & S & J \end{Bmatrix}$$

$$(-1)^{l_2+l_3}\,\langle n_{13}\, l_{13}\,,\ n_2\, l_2\ :\ L\,|\, n_{12}\, l_{12}\,,\ n_3\, l_3\ :\ L\rangle_d \quad \leftarrow \text{HO (Tamil-Moschinsky) bracket}$$

$$(-1)^{S_{13}+s_2+S_{12}+s_3}\,\hat{S}_{13}\,\hat{S}_{12} \begin{Bmatrix} s_2 & s_1 & S_{12} \\ s_3 & S & S_{13} \end{Bmatrix} (-1)^{T_{13}+t_2+T_{12}+t_3}\,\hat{T}_{13}\,\hat{T}_{12} \begin{Bmatrix} t_2 & t_1 & T_{12} \\ t_3 & T & T_{13} \end{Bmatrix}$$

$\hat{J}_{13} = \sqrt{2J_{13}+1}$

S. Liebig et al. EPJA (2017) arXiv:1510.06070v1

- Evaluation of 3N matrix elements:

$$\langle \alpha_3' | V_{3N} | \alpha_3\rangle = \langle \alpha_3' | \alpha_3'^{*(1)}\rangle\ \underbrace{\langle \alpha_3'^{*(1)} | V_{3N} | \alpha_3^{*(1)}\rangle}\langle \alpha_3^{*(1)} | \alpha_3\rangle$$

similar to $\langle \alpha_{12} | V_{NN} | \alpha_{12}'\rangle$

Jacobi NCSM basis

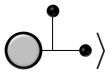
- A-body Jacobi HO basis: $|\alpha_A(NJT)\rangle \equiv$  constructed from (A-1) basis states

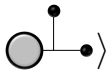
$$|\alpha_A^{*(1)}\rangle = |NJT\,\alpha_{A-1}\,n_A(l_A,s_A)I_A,t_A;\,(J_{A-1}I_A)J,(T_{A-1}t_A)T\rangle \equiv |$$ 

antisym. state of (A-1) $\equiv |$  $\rangle_{A-1}$

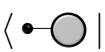
- diagonalize A-body antisymmetrizer to obtain antisymmetric states:

$$\langle$$  $|\,\frac{1}{A}(1-(A-1)\mathscr{P}_{A-1,A})\,|$  $\rangle\,\langle$ $|$ $\rangle_A = \lambda\,\langle$ $|$ $\rangle_A \qquad \lambda = 1$ (for certain (NJT) block)

  - expand $|$  $\rangle$ in a complete basis set : $|(\alpha^{*(1)})^{*(1)}\rangle \equiv$ 

$$|$$ $\rangle = \sum_{|$ $\rangle}$ $\langle$  $|$ $\rangle$  $= \sum_{|$  $\rangle} \delta_{spectator}$ $\langle$  $|$  $\rangle_{A-1}$ 

cfp(A-1), HO-ω independent

$$\langle$$  $|\;\mathscr{P}_{A-1,A}\;|$  $\rangle = \langle$ $|$  $\rangle\langle$  $|\;\mathscr{P}_{A-1,A}\;|$  $\rangle\langle$  $|$ $\rangle$

orth. transformation

⇒ totally antisymmetrized A-body basis state

$$|$$  $\rangle = |\alpha_A\rangle \equiv |iNJT\rangle = \sum_{\alpha_A^{*(1)}} \langle\alpha_A^{*(1)}|\alpha_A\rangle_{\rm cfp}\;|\alpha_A^{*(1)}\rangle$

HO-ω independent

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$$|$$ $\rangle = \sum_{|}$  $\rangle$ $\langle$  $|$ $\rangle\,|$  $\rangle = \sum_{|}$  $\rangle\,\delta_{spectator}\,\langle$  $|$  $\rangle_{A-1}\,|$  $\rangle$

cfp(A-1), HO-ω independent

$$\langle$$ $|\,\mathscr{P}_{A-1,A}\,|$ $\rangle = \langle$ $|$ $\rangle\,\langle$  $|\,\mathscr{P}_{A-1,A}\,|$  $\rangle\,\langle$  $|$ $\rangle$

orth. transformation

$\Rightarrow$ totally antisymmetrized A-body basis state

$$|\,\text{○}\,\rangle = |\alpha_A\rangle \equiv |iNJT\rangle = \sum_{\alpha_A^{*(1)}} \langle\alpha_A^{*(1)}|\alpha_A\rangle_{\text{cfp}}\;|\alpha_A^{*(1)}\rangle$$

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cfp(A-1), HO-ω independent

$$\rightarrow\quad \langle$$ $|\ \mathscr{P}_{A-1,A}\ |$ $\rangle = \langle$ $|$ $\rangle\,\langle$ $|\ \mathscr{P}_{A-1,A}\ |$ $\rangle\,\langle$ $|$ $\rangle$

orth. transformation

$\Rightarrow$ totally antisymmetrized A-body basis state

$$|$$ $\rangle = |\alpha_A\rangle \equiv |iNJT\rangle = \sum_{\alpha_A^{*(1)}} \langle\alpha_A^{*(1)}|\alpha_A\rangle_{\rm cfp}\ |\alpha_A^{*(1)}\rangle$

HO-ω independent

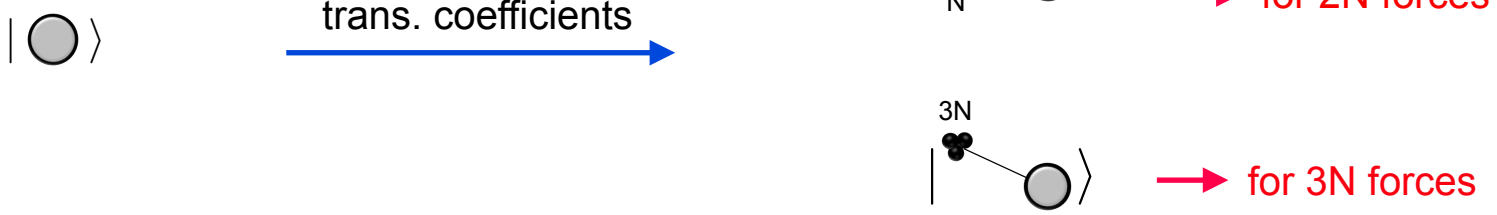
- basis truncation: $N = \mathscr{N}_{A-1} + 2n_A + l_A \le N_{max} \Rightarrow E_b = E_b(\omega, N_{max})$

require $N_{max} \to \infty$ extrapolation

Hamiltonian matrix elements in Jacobi basis

$$H_{rel} = \sum_{i<j=1}^{A} \left(\frac{2}{A}\frac{\mathbf{p}_{ij}^2}{m} + V_{ij}\right) + \sum_{i<j<k=1}^{A} V_{ijk} = \sum_{i<j=1}^{A} h_{ij}^{NN} + \sum_{i<j<k=1}^{A} V_{ijk}, \qquad \langle \bigcirc | H_{rel} | \bigcirc \rangle$$

- transformation to other bases



$$\Rightarrow \langle \bigcirc | \sum_{i<j=1}^{A} h_{ij}^{NN} | \bigcirc \rangle = \langle \bigcirc | \cdot \rangle \langle \cdot | \sum_{i<j=1}^{A} h_{ij}^{NN} | \cdot \rangle \langle \cdot | \bigcirc \rangle$$

$$= \langle \bigcirc | \cdot \rangle \; \delta_{core(A-2)} \binom{A}{2} \underbrace{\langle \alpha_{12} | h_{ij}^{NN} | \alpha'_{12} \rangle}_{\text{2-body matrix element}} \langle \cdot | \bigcirc \rangle$$

arXiv:1510.06070v1

- transition coefficients:

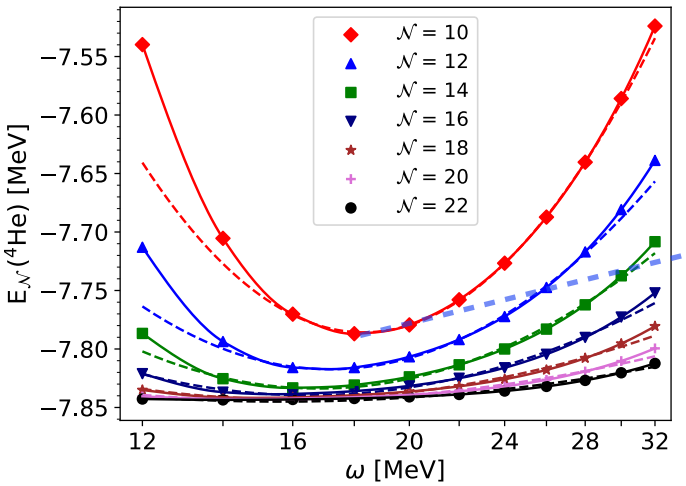
$$\langle \bigcirc | \cdot \rangle = \langle \underbrace{\bigcirc | \cdot}_{\text{cfp(A)}} \rangle \langle \underbrace{\cdot | \cdot}_{\delta_{spec}\ \text{cfp(A-1)}} \rangle \langle \underbrace{\cdot | \cdot}_{\text{orth. trans.}} \rangle \quad \rightarrow \text{HO-}\omega \text{ independent}$$

→ transition coefficients + cfp are prepared for each NJT block separately and stored in HDF5 format that allows an efficient parallel input/output

Extrapolation of binding energies

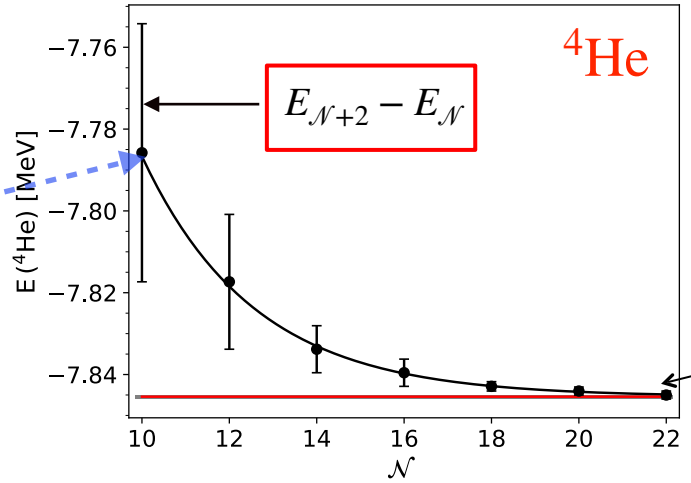
- Two parameters in NCSM: HO-ω, and $\mathcal{N}$ $\Rightarrow$ $E_b = E_b(\omega, \mathcal{N})$

$$E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$$



$E_{\mathcal{N}}$

$$E_{\mathcal{N}} = E_\infty + Ae^{-b\mathcal{N}}$$



NN : χN^{4}LO + (500)
$\lambda = 2.0\,\mathrm{fm}^{-1}$

$\delta E = E_\infty - E_{\mathcal{N}_{max}}$

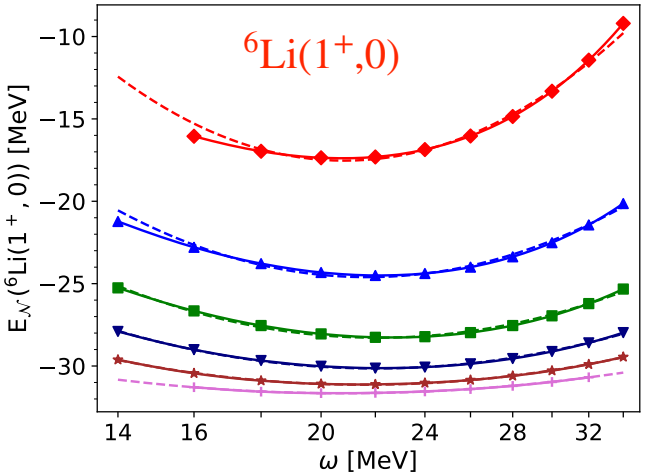
- ω_{opt} shifts to smaller values as $\mathcal{N}$ increases
- ω-dependence energy curves flatten with increasing $\mathcal{N}$

- $E_{\mathcal{N}}$ converges to E_∞ strictly from above

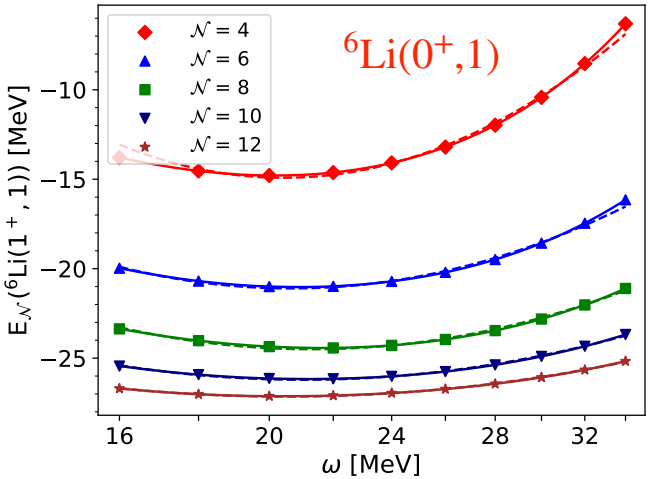
Extrapolation of excitation energies

- Two parameters in NCSM: HO-ω, and $\mathcal{N}$ $\Rightarrow$ $E_b = E_b(\omega, \mathcal{N})$
  - ω_{opt} for ground and excited states are not necessarily the same

$$E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$$



$$E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$$

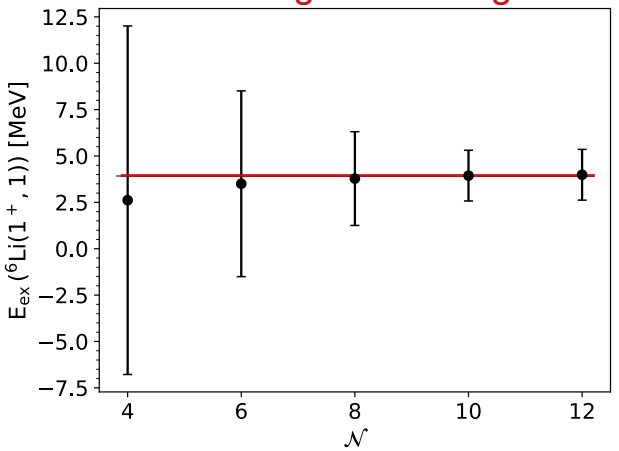


NN : χN^{4}LO + (500)
$\lambda = 2.0\,\mathrm{fm}^{-1}$

→ perform $\mathcal{N}$-space extrapolation on $E_{ex,\mathcal{N}}$

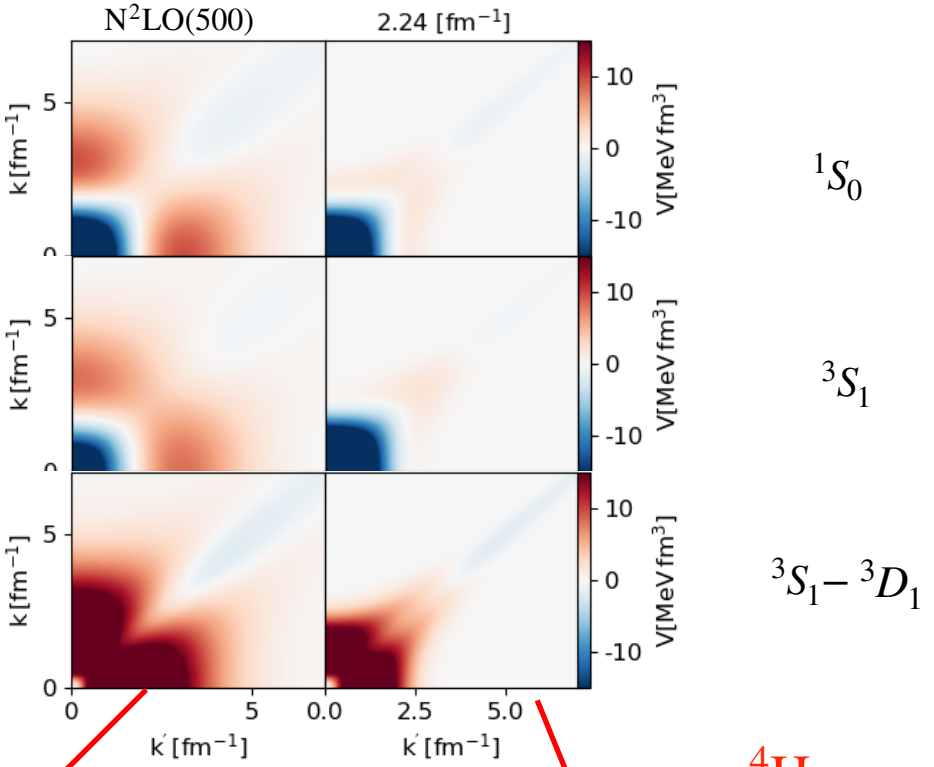
$$E_{ex,\mathcal{N}}(1^+,1) = E_{\mathcal{N}}(1^+,1) - E_{\mathcal{N}}(1^+,0)$$

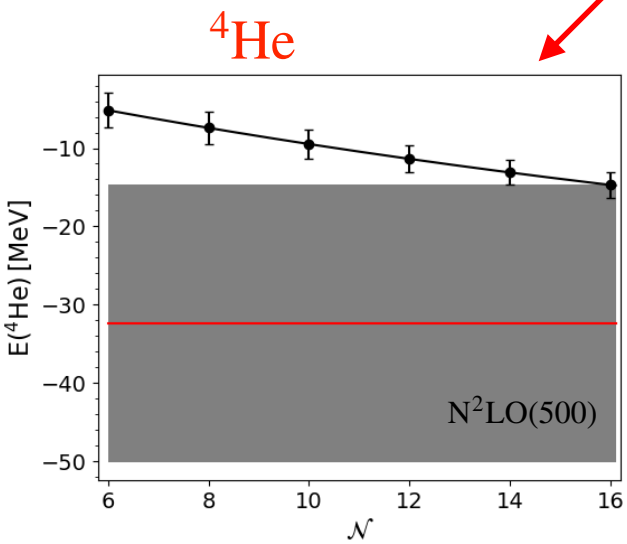
weighted average

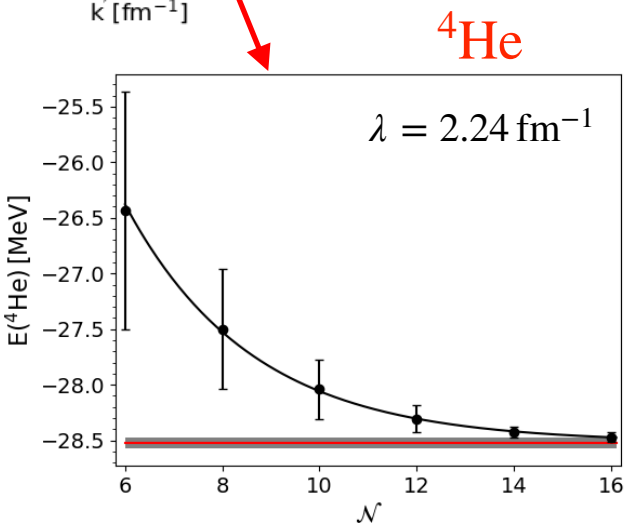


Convergence of E with respect to $\mathcal{N}$

low-energy observables (long-wavelength information) converge slowly due strong couplings of low- and high-momentum states



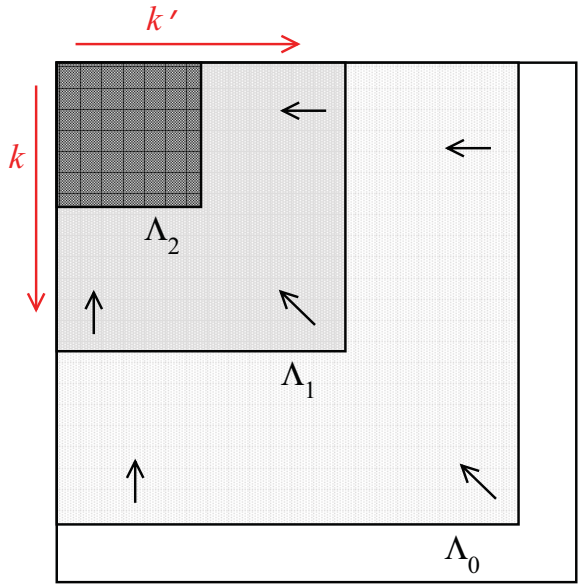




Renormalization Group (RG) methods

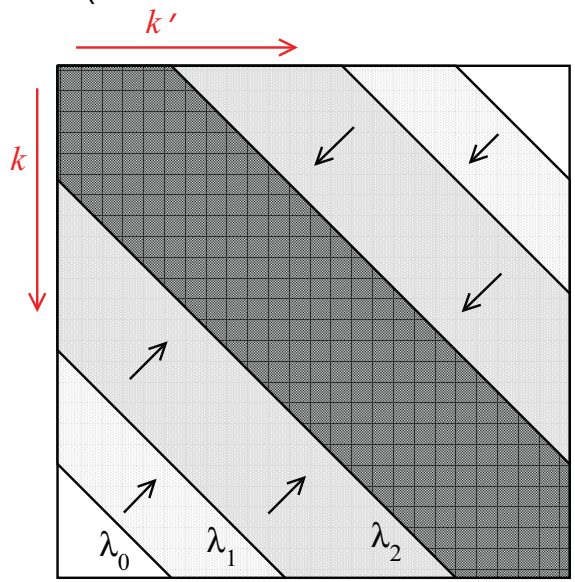
Idea: use RG methods to decouple low- and high-energy degrees of freedom

- $V_{low\ k}$ transformation
- Similarity renormalization group (SRG) evolution approach



$V_{low\ k}$ transformation

(R.J. Furnstahl et al. NPBP 228 (2012))



SRG evolution

→ $V_{low\ k}$ is phase equivalent to V_{SRG} at low momentum

→ SRG technique is much simpler + inclusion of higher body forces is straightforward

Similarity Renormalization Group (SRG)

Idea: continuously apply unitary transformation to H to suppress off-diagonal matrix elements

→ observables are conserved due to unitarity of transformation ✓

F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$\frac{dV(s)}{ds} = \left[\left[T_{rel}, V(s)\right], H(s)\right], \qquad H(s) = T_{rel} + V(s); \qquad T_{rel} = T_{12} + T_3 = T_{23} + T_1 = T_{31} + T_2$$

$$V(s) = V_{12}(s) + V_{13}(s) + V_{23}(s) + V_{123}(s) + \ldots$$

- separate SRG flow equations for 2N and 3N interactions:

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= [[T_{ij}, V_{ij}], T_{ij} + V_{ij}], \\
\frac{dV_{123}}{ds} &= [[T_{12}, V_{12}], V_{31} + V_{23} + V_{123}] \\
&+ [[T_{31}, V_{31}], V_{12} + V_{23} + V_{123}] \\
&+ [[T_{23}, V_{23}], V_{12} + V_{31} + V_{123}] \\
&+ [[T_{\rm rel}, V_{123}], H_s] \ .
\end{aligned}$$

Eqs.(1)

⇒ no disconnected terms in $\frac{dV_{123}}{ds}$: dangerous delta functions are cancelled

- Eqs.(1) are solved by projecting on a partial-wave decomposed Jacobi-momentum basis

SRG flow equations

- SRG flow equation for 2N potential: $|p\,\alpha_{12}\rangle \equiv |p,(l_{12}s_{12})J_{12}\,(t_1t_2)t_{12}\,m_{t12}\rangle$

$$\frac{dV_{12}^{\alpha_{12}\alpha'_{12}}(pp')}{ds} = \underbrace{\left\{T_{12}^{\alpha_{12}}(p)\frac{p'^2}{2\mu^{\alpha'_{12}}} + T_{12}^{\alpha'_{12}}(p')\frac{p^2}{2\mu^{\alpha_{12}}} - T_{12}^{\alpha_{12}}(p)\frac{p^2}{2\mu^{\alpha_{12}}} - T_{12}^{\alpha'}(p')\frac{p'^2}{2\mu^{\alpha'_{12}}}\right\}}_{\text{drives } V_{12} \text{ towards the}} V_{12}^{\alpha_{12}\alpha'_{12}}(pp')$$

$$+\sum_{\tilde{\alpha_{12}}}\underbrace{\int_0^\infty dk k^2 \left\{\frac{p^2}{2\mu^{\alpha_{12}}} + \frac{p'^2}{2\mu^{\alpha'_{12}}} - \frac{k^2}{\mu^{\tilde{\alpha_{12}}}}\right\} V_{12}^{\alpha_{12}\tilde{\alpha_{12}}}(pk)V_{12}^{\tilde{\alpha_{12}}\alpha'_{12}}(kp')}_{\text{preserves unitarity}}$$

Eq.(2)

- SRG flow equation for 3N interactions: $|pq\,\alpha\rangle \equiv |pq,((l_{12}s_{12})J_{12}\;(l_3s_3)I_3)J\;((t_1t_2)T_{12}\,t_3)T\rangle$

$$\langle p'q'\alpha'|\frac{dV_{123}}{ds}|pq\alpha\rangle = \frac{2}{3}\langle p'q'\alpha'|(1+P)([T_{12},V_{12})]P_{12}P_{23}V_{12} - V_{12}P_{12}P_{23}[T_{12},V_{12}])(1+P)|pq\alpha\rangle$$

$$+\frac{1}{3}\langle p'q'\alpha'|(1+P)([T_{12},V_{12})]V_{123} - V_{123}[T_{12},V_{12}])(1+P)|pq\alpha\rangle$$

$$+\frac{1}{9}\langle p'q'\alpha'|(1+P)([T_{123},V_{123}]T_{123} - T_{123}[T_{123},V_{123}])(1+P)|pq\alpha\rangle$$

$$+\frac{1}{9}\langle p'q'\alpha'|(1+P)([T_{123},V_{123}]V_{12} - V_{12}[T_{123},V_{123}])(1+P)|pq\alpha\rangle$$

$$+\frac{2}{9}\langle p'q'\alpha'|(1+P)([T_{123},V_{123}]P_{12}P_{23}V_{12} - V_{12}P_{12}P_{23}[T_{123},V_{123}])(1+P)|pq\alpha\rangle$$

$$+\frac{1}{9}\langle p'q'\alpha'|(1+P)([T_{123},V_{123}]V_{123} - V_{123}[T_{123},V_{123}])(1+P)|pq\alpha\rangle$$

$$P = P_{12}P_{23} + P_{23}P_{13}$$

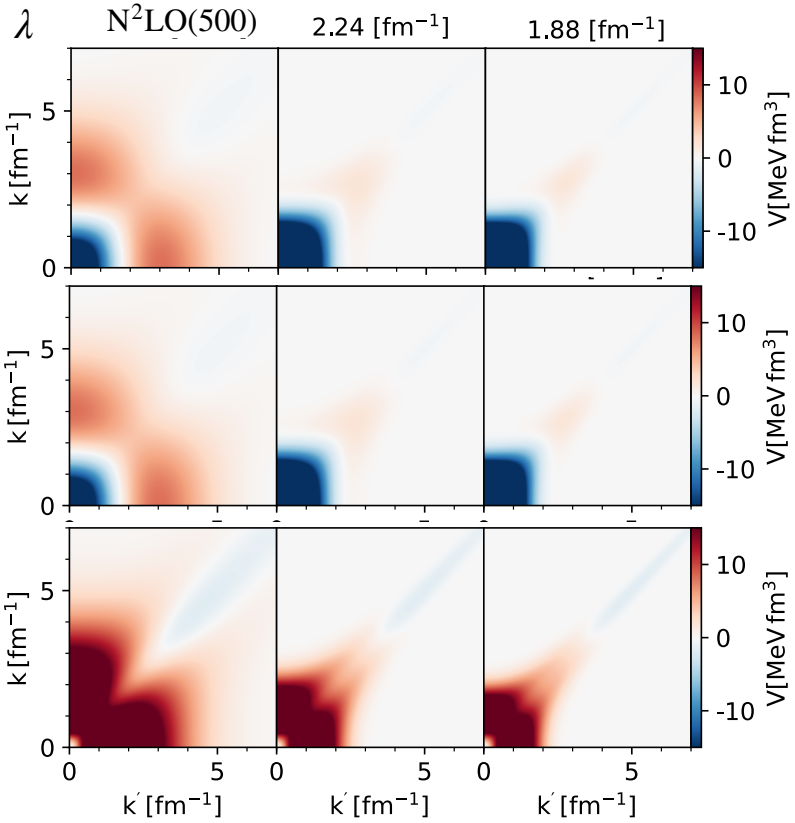
Eq.(3)

- SRG evolution is only approximately unitary if higher-body forces are omitted ✓

SRG evolution of NN χN^{2}LO(500)

- $\lambda = (4\mu^2/s)^{1/4}$, $[\lambda] = [p]$: $\lambda \sim$ width of the band-diagonal structure of V_{NN} in p-space

(S.K. Bogner et al. PRC 75 (2007))



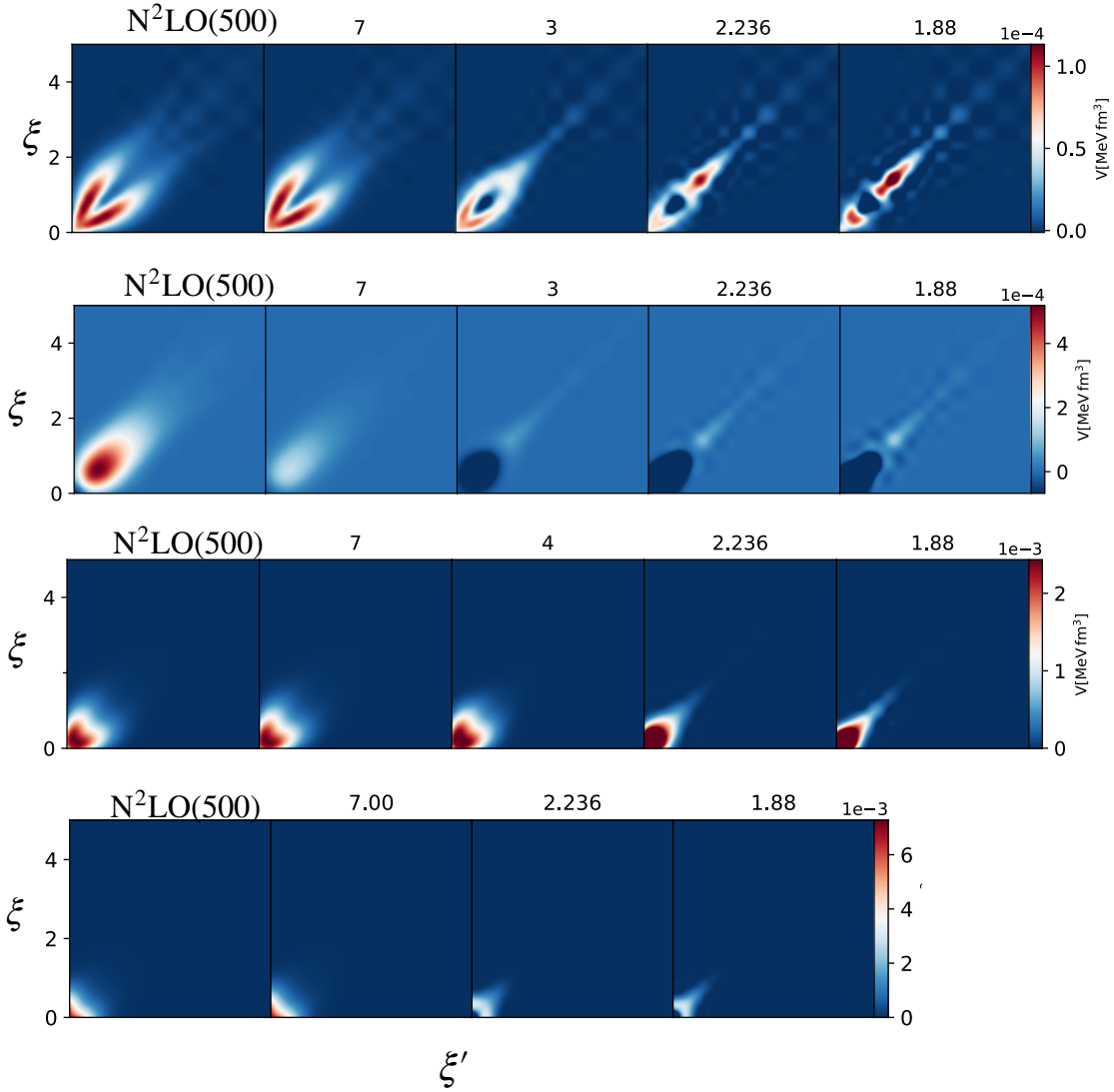
$^1S_0$

$^3S_1$

$^3S_1 - {}^3D_1$

SRG evolution of $V_{123}(pq\alpha, p'q'\alpha')$ χN^{2}LO(500)

- hyperradius $\xi^2 = p^2 + \frac{3}{4}q^2, \qquad tan\theta = \frac{2p}{\sqrt{3}q} = \frac{\pi}{12}, \qquad \alpha = \alpha' = 1 \quad \Rightarrow \quad V_{123} = V_{123}(\xi', \xi)$



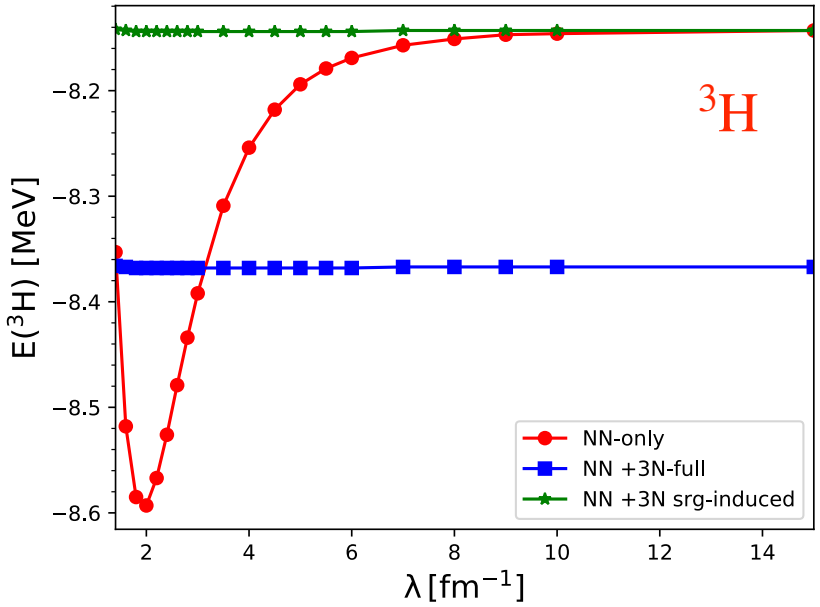
$(J^\pi, T) = (9/2^+, 1)$

$(J^\pi, T) = (7/2^+, 1)$

$(J^\pi, T) = (5/2^+, 1)$

$(J^\pi, T) = (1/2^+, 1)$

$E(^3H)$ with $\chi N^2LO(500)$



3N: $N^2LO(500)$

$c_D = -1.28,\ c_E = -0.38$

- SRG is approximately unitary if higher-body forces are omitted
- SRG-induced 3N forces are comparable to the initial (bare) 3N forces

Results for A=3-6

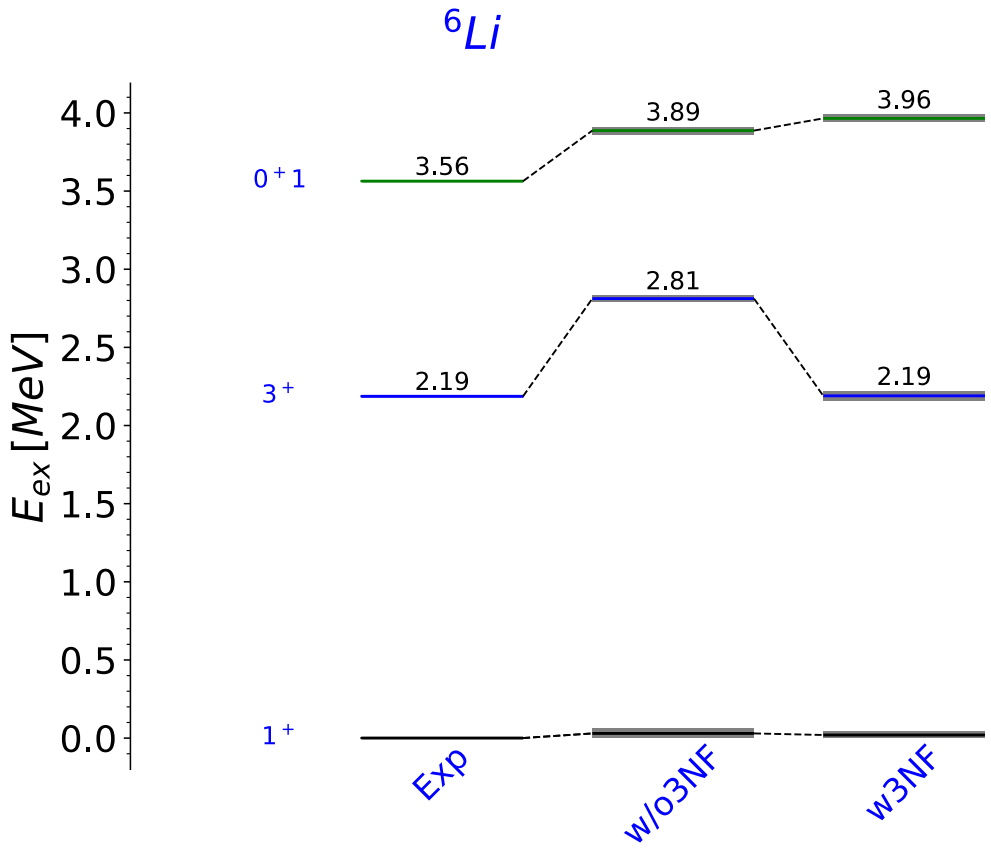
3N: $N^2LO(500)$

$c_D = -1.63,\ c_E = -0.063$

	$N^2LO(500) + 3N$		$N^2LO(500)$*	Exp.
	J−NCSM	F−Y		
^3H	−8.477	−8.482	−7.92	−8.482
^4He	−28.57	−28.72	−25.85	−28.296
^6Li	−32.19		−28.77	−31.99

* P. Maris et. al., PRC 103. 054001

Energy spectrum of ^{6}Li



2N+ 3N: $N^2LO(500)$

$c_D = -1.63,\ c_E = -0.063$

→ 2N+ 3N at N^2LO give a good prediction for $(3^+,0)$ state.

Summary

- Jacobi NCSM approach:
  - Construction Jacobi basis state, intermediate bases for 2N + 3N forces
  - Extrapolation of binding and excitation energies
- Similarity Renormalisation Group (SRG):
  - Construct flow equations for 2N +3N forces
  - Study SRG evolution of 2N +3N at $\mathrm{N^2LO(500)}$
- NCSM results for A=4-6 nuclei

Questions?