Notes for IAV model

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1 The problem

1.1 The reaction

The IAV model deals with the reaction which takes the form:

$$a \ + \ A \rightarrow b \ + \text{anything} \tag{1}$$

where a is a bound state consists of x and b (x+b). We call b the spectator because the experiment detects it. 'Anything' here can be seen as x+A, but there are a few channels:

1. (x+A) stays in the ground state, which is called the elastic reaction
2. (x+A) goes to excited state, which is called the inelastic reaction
3. Particle trnasfer may occur between x and A

1.2 The post-form and prior-form

Austern and Vincent (AV) and Kasano and Ichimura (KI) develop the post-form formula of the inclusive breakup, while Li, Udagawa and Tamura (LUT) derive a similar prior-form formula of the reaction. An important task is to show how these methods can be transformed from one to another. And how approximations are taken carefully to achieve a promising result.

2 Post-form derivation

2.1 The model Hamiltonian

The Hamiltonian is written as:

$$H = H_A(\xi) + K_b + K_x + V_{xA} + U_b + V_{bx} \tag{2}$$

H_A is the internal Hamiltonian of the target nucleus A and ξ stands for its internal coordinate. K_b and K_x are the kinetic operator of b and x. V_{xA}, V_{bx} and V_{bA} are the interacting potentials. U_b stands for the optical potential which replaces V_{bA}. In this model we don't care about the internal state of x and b.

2.2 Notations of the states

The target nucleus ground state wave function is written as Φ_A with energy E_A:

$$H_A \Phi_A = E_A \Phi_A \tag{3}$$

The eigenstate of the system x+A is written as Ψ^c_{xA}, where the superscript c represents different channels:

$$H_{xA}\Psi^c_{xA} = (H_A + K_x + V_{xA})\Psi^c_{xA} = E^c \Psi^c_{xA} \tag{4}$$

The internal state of the projectile is denoted with $\phi_a(\boldsymbol{r_{bx}})$ with energy E_a, where $\boldsymbol{r_{bx}} = \boldsymbol{r_b} - \boldsymbol{r_x}$.

The optical scattering wave functions are defined as:

$$[K_a + U_a(\boldsymbol{r_a})]\chi_a^{(+)}(\boldsymbol{r_a}) = (E - E_A - E_a)\chi_a^{(+)}(\boldsymbol{r_a}) \tag{5}$$

$$[K_b + U_b^\dagger(\boldsymbol{r_b})]\chi_b^{(-)}(\boldsymbol{r_b}) = E_b \chi_b^{(-)}(\boldsymbol{r_b}) \tag{6}$$

E_A is the total energy of the target nucleus A, and E_a is the total energy of the projectile a. E_b is the relative kinetic energy between b and B. $\chi_a^{(+)}(\boldsymbol{r_a})$ is the initial scattering state between a and A. $\chi_b^{(-)}(\boldsymbol{r_b})$ is the final scattering state between b and B.

2.3 Cross section

The general post-form DWBA expression for the inclusive breakup of the outgoing spectator b is:

$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}E_b}\right|_{\mathbf{post}} = \frac{2\pi}{\hbar v_a}\rho(E_b)\sum_c |\langle \chi_b^{(-)}\Psi^c_{xA}|V_{\mathrm{post}}|\chi_a^{(+)}\phi_a\Phi_A\rangle|^2 \delta(E - E_b - E^c) \tag{7}$$

where $|\chi_b^{(-)}\Psi^c_{xA}\rangle$ represents the final state and $|\chi_a^{(+)}\phi_a\Phi_A\rangle$ represents the initial state. This reveals the core idea of the DWBA approximation.

The delta function can be replaced with the principal value therom:

$$\delta(E - E_b - E^c) = \frac{-1}{\pi}\mathrm{Im}(E^+ - E_b - E^c)^{-1} \tag{8}$$

so the cross section reads:

$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}E_b}\right|_{\mathbf{post}} = \frac{-2}{\hbar v_a}\rho(E_b)\sum_c \langle \chi_a^{(+)}\phi_a\Phi_A|\chi_b^{(-)}\Psi_{xA}^c\rangle \times \mathrm{Im}(E^+ - E_b - E^c)^{-1}\langle \Psi_{xA}^c \chi_b^{(-)}|V_{\mathrm{post}}|\chi_a^{(+)}\phi_a\Phi_A\rangle \tag{9}$$

replace E^c with H_{xA} and then use the complete relationship:

$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}E_b}\right|_{\mathbf{post}} = \frac{-2}{\hbar v_a}\rho(E_b)\langle \chi_a^{(+)}\phi_a\Phi_A|\chi_b^{(-)}) \times \left[\sum_c \mathrm{Im}(E^+ - E_b - H_{xA})^{-1}|\Psi_{xA}^c\rangle\langle \Psi_{xA}^c|\right](\chi_b^{(-)}|V_{\mathrm{post}}|\chi_a^{(+)}\phi_a\Phi_A\rangle \tag{10}$$

Define the source term:

$$|\rho_b(\boldsymbol{r_b})\rangle = (\chi_b^{(-)}|V_{\mathrm{post}}|\chi_a^{(+)}\phi_a\Phi_A\rangle \tag{11}$$

which is a state vector in one subspace of the whole three-body Hilbert space.

Then carry out the optical reduction:

$$\begin{aligned}(\Psi_A|(E^+ - E_b - H_{xA})^{-1}|\Psi_A) &= (\Psi_A|(E^+ - E_b - H_A - K_x - V_{xA})^{-1}|\Psi_A)\\ &= (\Psi_A|(E_x^+ - K_x - V_{xA})^{-1}|\Psi_A)\\ &= (\Psi_A|(E_x^+ - K_x - U_x)^{-1}|\Psi_A)\\ &= (E_x^+ - K_x - U_x)^{-1}(\Psi_A|\Psi_A)\\ &= (E_x^+ - K_x - U_x)^{-1}\end{aligned} \tag{12}$$

where $E_x^+ = E^+ - E_b - E_A$. Finally, define the Green operator:

$$G_x \equiv (E_x^+ - K_x - U_x)^{-1} \tag{13}$$

we arrive at the final form:

$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}E_b}\right|_{\mathbf{post}} = \frac{-2}{\hbar v_a}\rho(E_b) \times \langle \rho_b(\boldsymbol{r_b})|\mathrm{Im}\ G_x|\rho_b(\boldsymbol{r_b})\rangle \tag{14}$$

2.4 Green's operator

Two Green's operator are defines as:

$$G_x = (E_x^+ - K_x - U_x)^{-1} \tag{15}$$

$$G_0 = (E_x^+ - K_x)^{-1} \tag{16}$$

They satisfy the following relation:

$$\begin{aligned}G_x &= G_0(1 + U_x G_x)\\ &= (1 + G_x^\dagger U_x^\dagger)G_0(1 + U_x G_x) - G_x^\dagger U_x^\dagger G_x\end{aligned} \tag{17}$$

$$G_x^\dagger = (1 + G_x^\dagger U_x^\dagger) G_0^\dagger (1 + U_x G_x) - G_x^\dagger U_x G_x \tag{18}$$

so the imaginary part reads:

$$\begin{aligned} \mathrm{Im} G_x &= \frac{1}{2i}(G_x - G_x^\dagger) \\ &= (1 + G_x^\dagger U_x^\dagger) \mathrm{Im} G_0 (1 + U_x G_x) + G_x^\dagger W_x G_x \end{aligned} \tag{19}$$

The two terms above seperate the cross section into two parts:

$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}E_b}\right|_{\mathbf{post}} = \left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}E_b}\right|_{\mathbf{post}}^{\mathbf{EB}} + \left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}E_b}\right|_{\mathbf{post}}^{\mathbf{BF}} \tag{20}$$