

# Quantum many-body dynamics in heavy-ion fusion reactions around the Coulomb barrier



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1. Nuclear Reactions: overview
2. Fusion of light nuclei and Feshbach resonances
3. Fusion of medium-heavy nuclei and quantum tunneling
4. Fusion for superheavy nuclei and open quantum systems
5. Microscopic modelling of low-energy nuclear reactions
6. Fission
7. Summary

# Introduction: low-energy nuclear physics

## □ behaviors of atomic nuclei as a quantum many-body systems

← understanding based on strong interaction

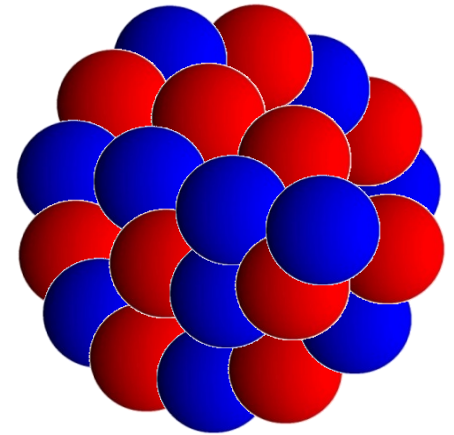
### ➤ static properties: nuclear structure

- ✓ ground state properties  
(mass, size, shape,...)
- ✓ excitations
- ✓ nuclear matter
- ✓ decays

### ➤ dynamics: nuclear reactions

**nucleus: a composite system**

- ✓ various sort of reactions



- elastic scattering
- inelastic scattering
- transfer reaction
- breakup reactions
- fusion reactions

# Introduction: low-energy nuclear physics

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← understanding based on strong interaction

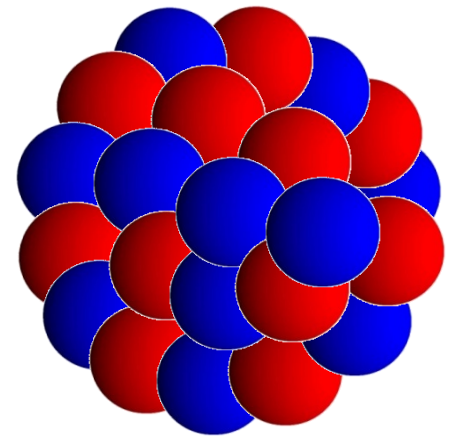
### ➤ static properties: nuclear structure

- ✓ ground state properties  
(mass, size, **shape,...**)
- ✓ **excitations**
- ✓ nuclear matter
- ✓ decays

### ➤ dynamics: nuclear reactions

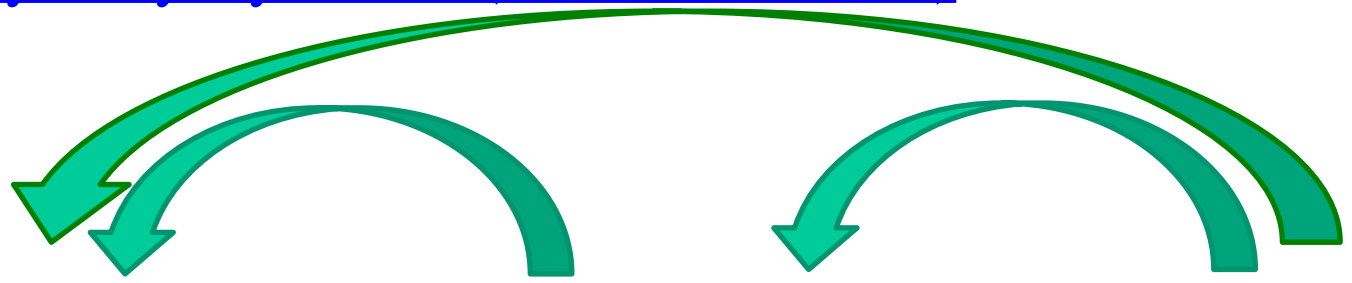
**nucleus: a composite system**

- ✓ various sort of reactions
- ✓ **an interplay between nuclear structure and reaction**



- elastic scattering
- inelastic scattering
- transfer reaction
- breakup reactions
- fusion reactions

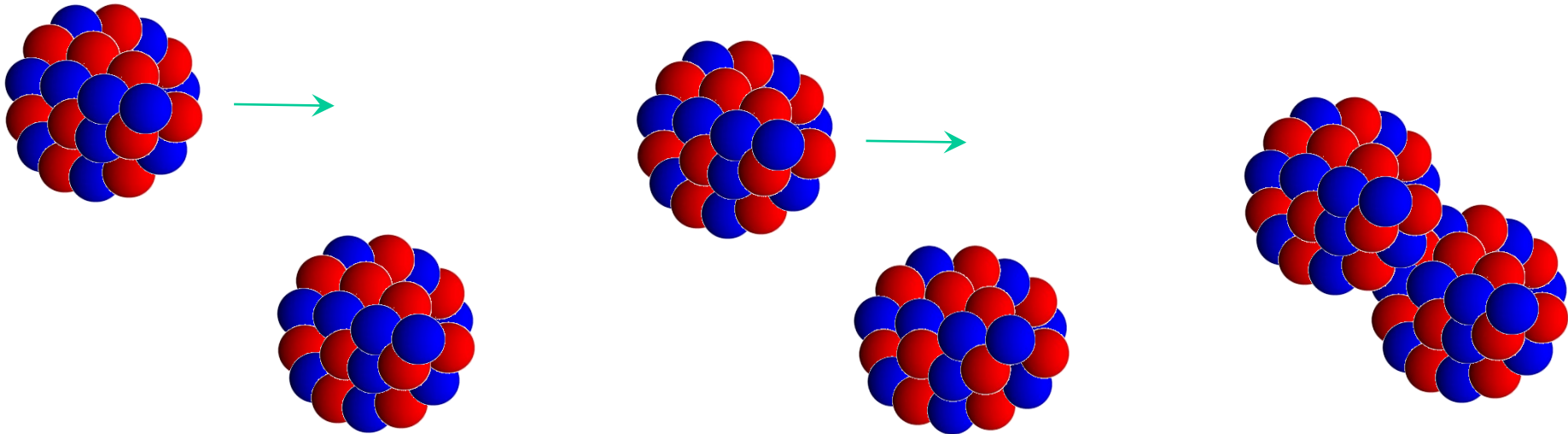
# Quantum Many-body Dynamics (nuclear reactions)



elastic scattering

inel. scattering

fusion

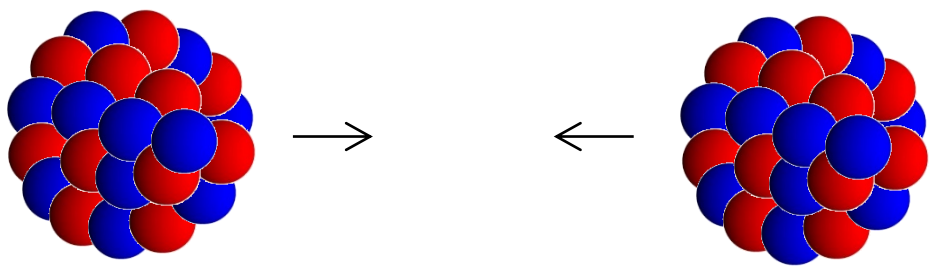


a quantum reaction theory  
to describe several reaction processes  
simultaneously



# Coupled-channels method: a quantal scattering theory with excitations

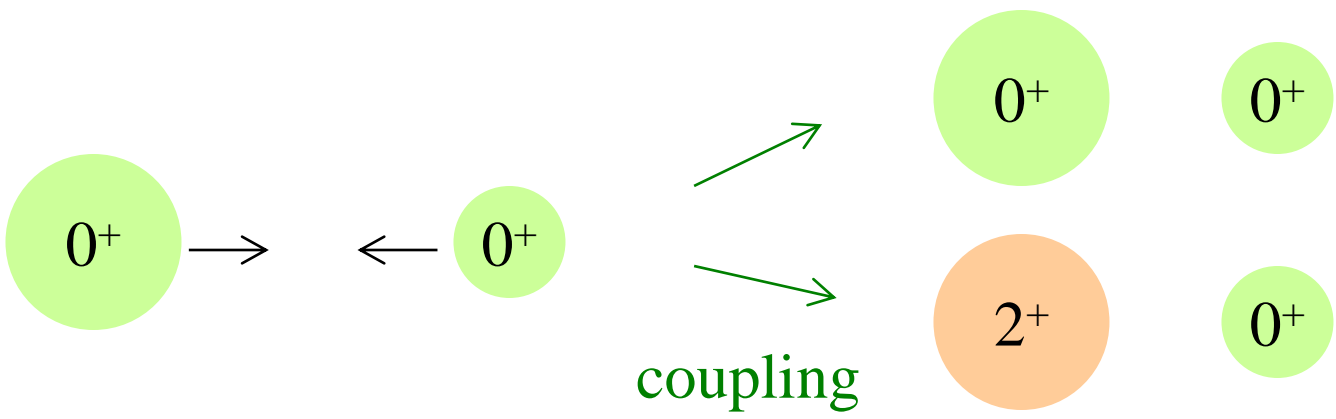
many-body problem



still very challenging

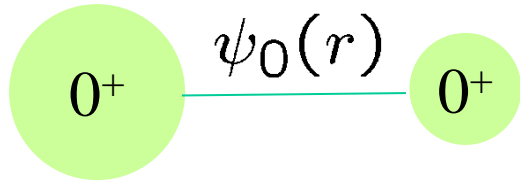


two-body problem, but with excitations  
(the coupled-channels approach)



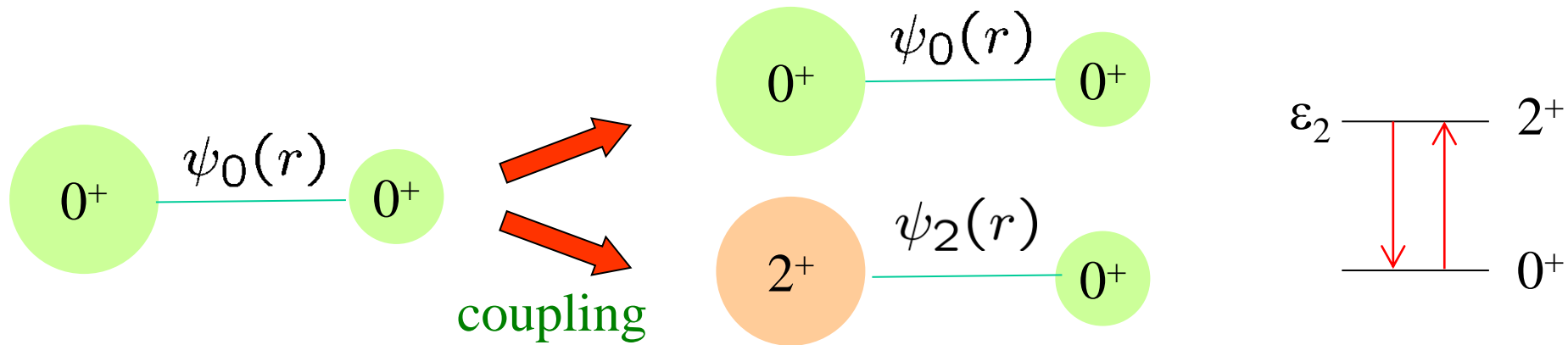
scattering theory with excitations

# Coupled-channels method: a quantal scattering theory with excitations



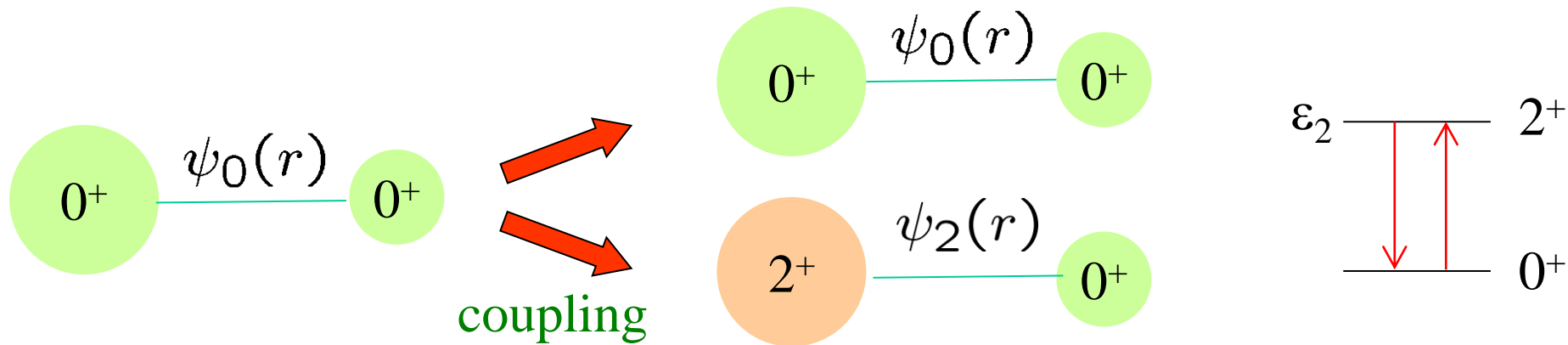
$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(\mathbf{r}) - E \right] \psi_0(\mathbf{r}) = 0$$

# Coupled-channels method: a quantal scattering theory with excitations



$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) - E \right] \psi_0(\mathbf{r}) = -F_{0 \rightarrow 2}(r) \psi_2(\mathbf{r})$$

# Coupled-channels method: a quantal scattering theory with excitations



$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) - E \right] \psi_0(\mathbf{r}) = -F_{0 \rightarrow 2}(r) \psi_2(\mathbf{r})$$

$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_2(r) - (E - \epsilon_2) \right] \psi_2(\mathbf{r}) = -F_{2 \rightarrow 0}(r) \psi_0(\mathbf{r})$$

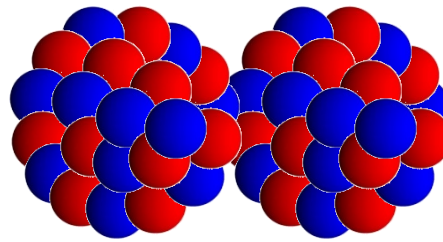
- Fusion  $\rightarrow$  an absorbing potential (an optical potential)
- excitations to unbound states  $\rightarrow$  breakup reactions  
(neutron-rich nuclei)

a recent review: K. Hagino, K. Ogata, and A.M. Moro, PPNP in press.

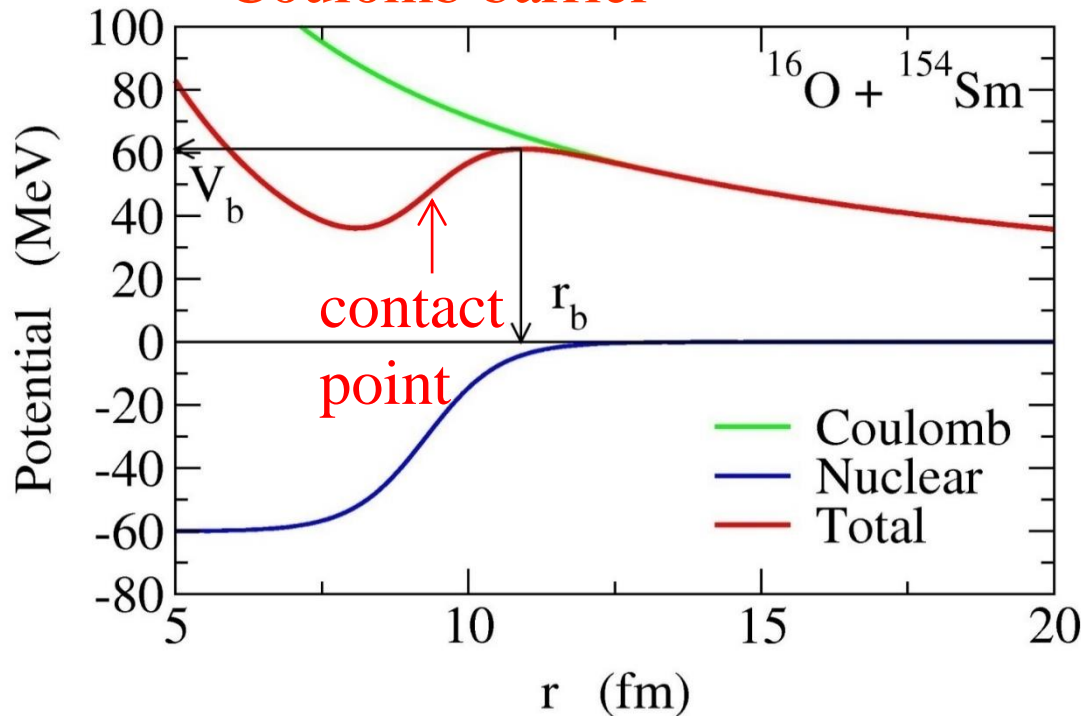
arXiv: 2201.09512



# Fusion Reactions



Coulomb barrier



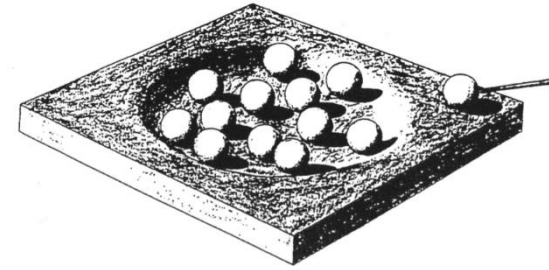
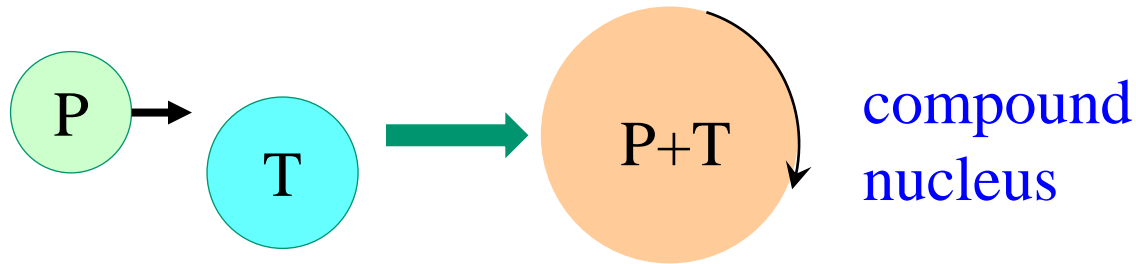
- Nuclear interaction (attractive)
- Coulomb interaction (repulsive)



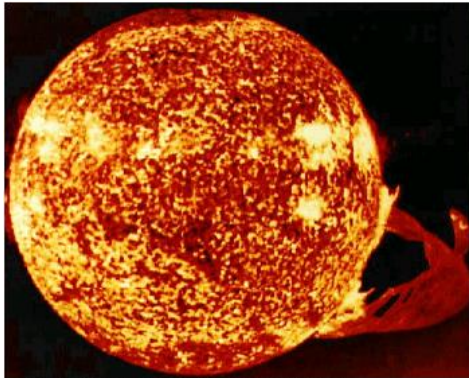
potential barrier

Fusion reactions → a many-body quantum tunneling

# Fusion Reactions

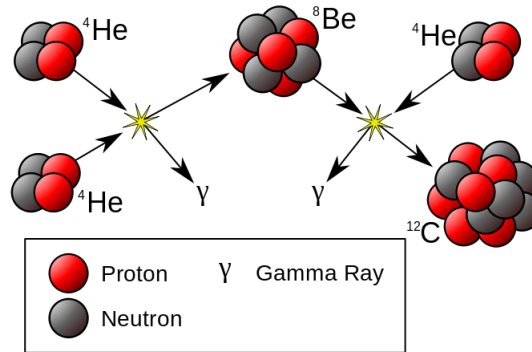


cf. Bohr '36

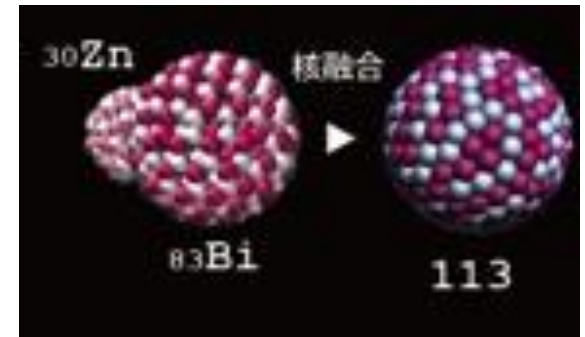


NASA, Skylab space station on December 19, 1973, solar flare reaching 588 000 km off solar surface

energy production  
in stars (Bethe '39)



nucleosynthesis

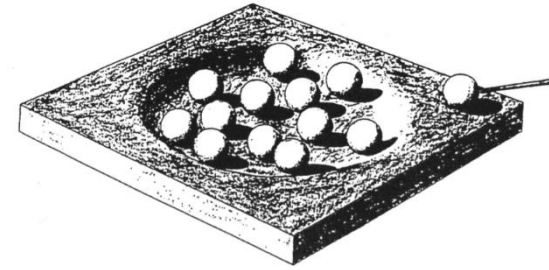
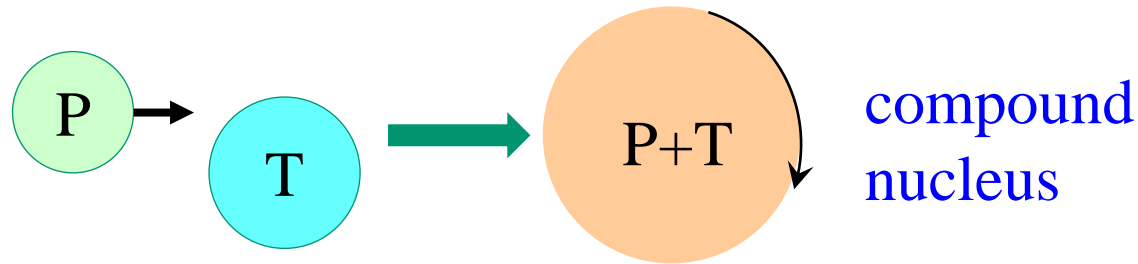


superheavy elements

**Fusion and fission:** large amplitude motions of quantum many-body systems with strong interaction

← microscopic understanding: **an ultimate goal of nuclear physics**

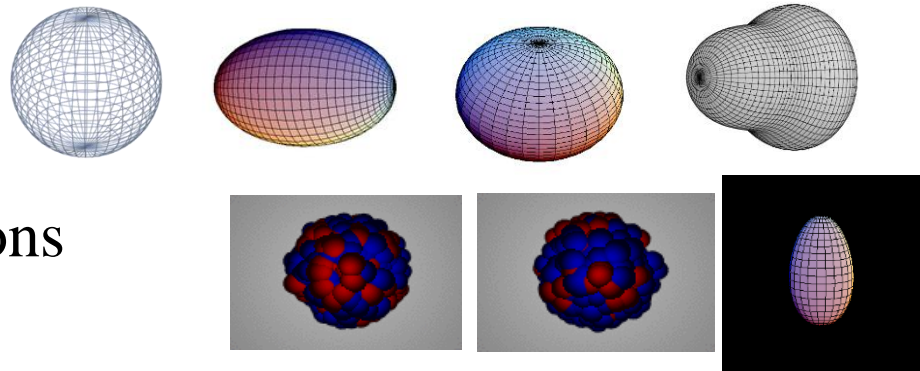
# Fusion Reactions



cf. Bohr '36

## ✓ Many-particle tunneling

- rich intrinsic motions
  - several nuclear shapes
  - several surface vibrations

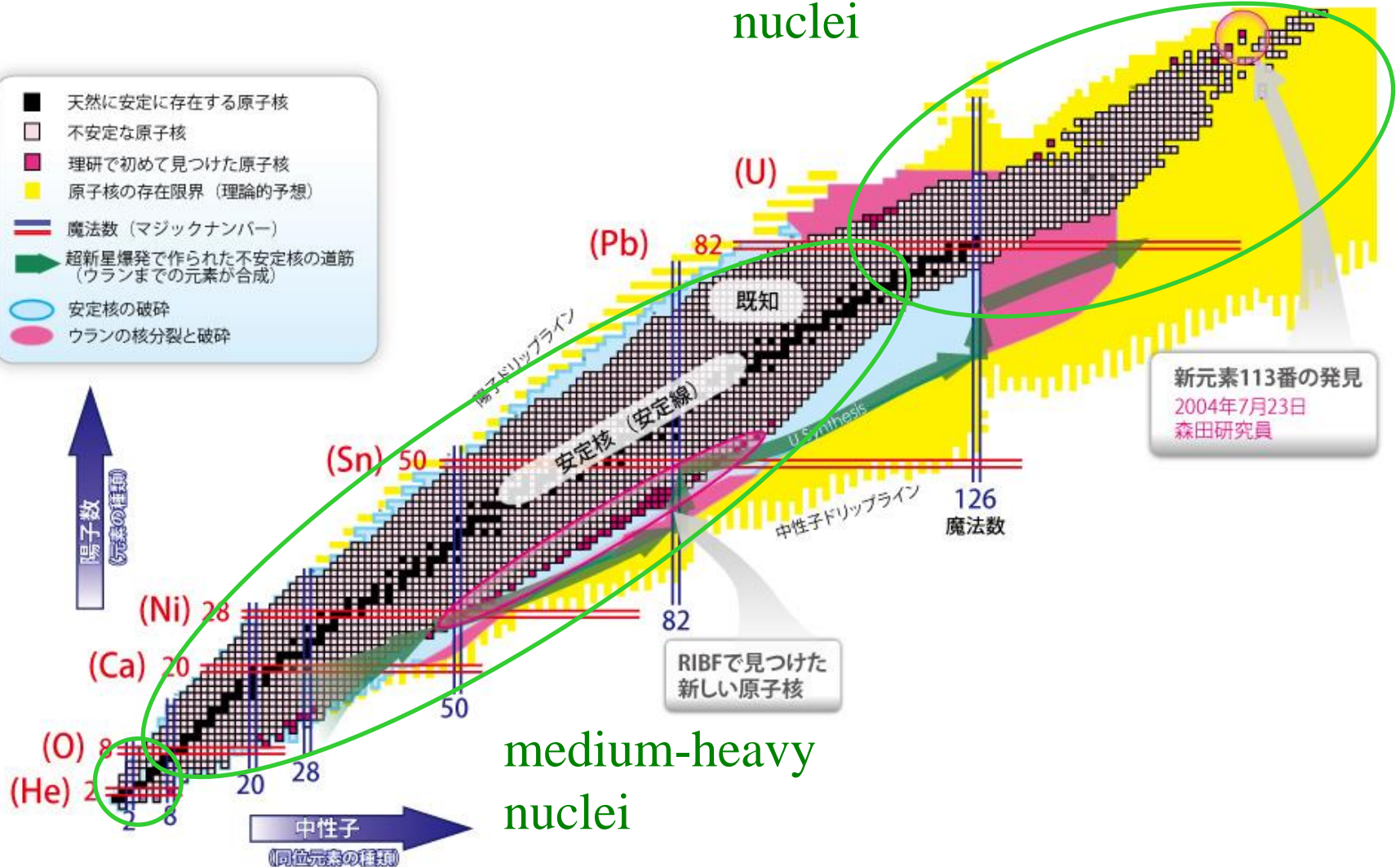


several modes  
and adiabaticities

H.I. fusion reaction = an ideal playground to study quantum tunneling with many degrees of freedom

# Heavy and Superheavy nuclei

- 天然に安定に存在する原子核
- 不安定な原子核
- 理研で初めて見つけた原子核
- 原子核の存在限界 (理論的予想)
- 魔法数 (マジックナンバー)
- ➔ 超新星爆発で作られた不安定核の道筋 (ウランまでの元素が合成)
- 安定核の破碎
- ウランの核分裂と破碎

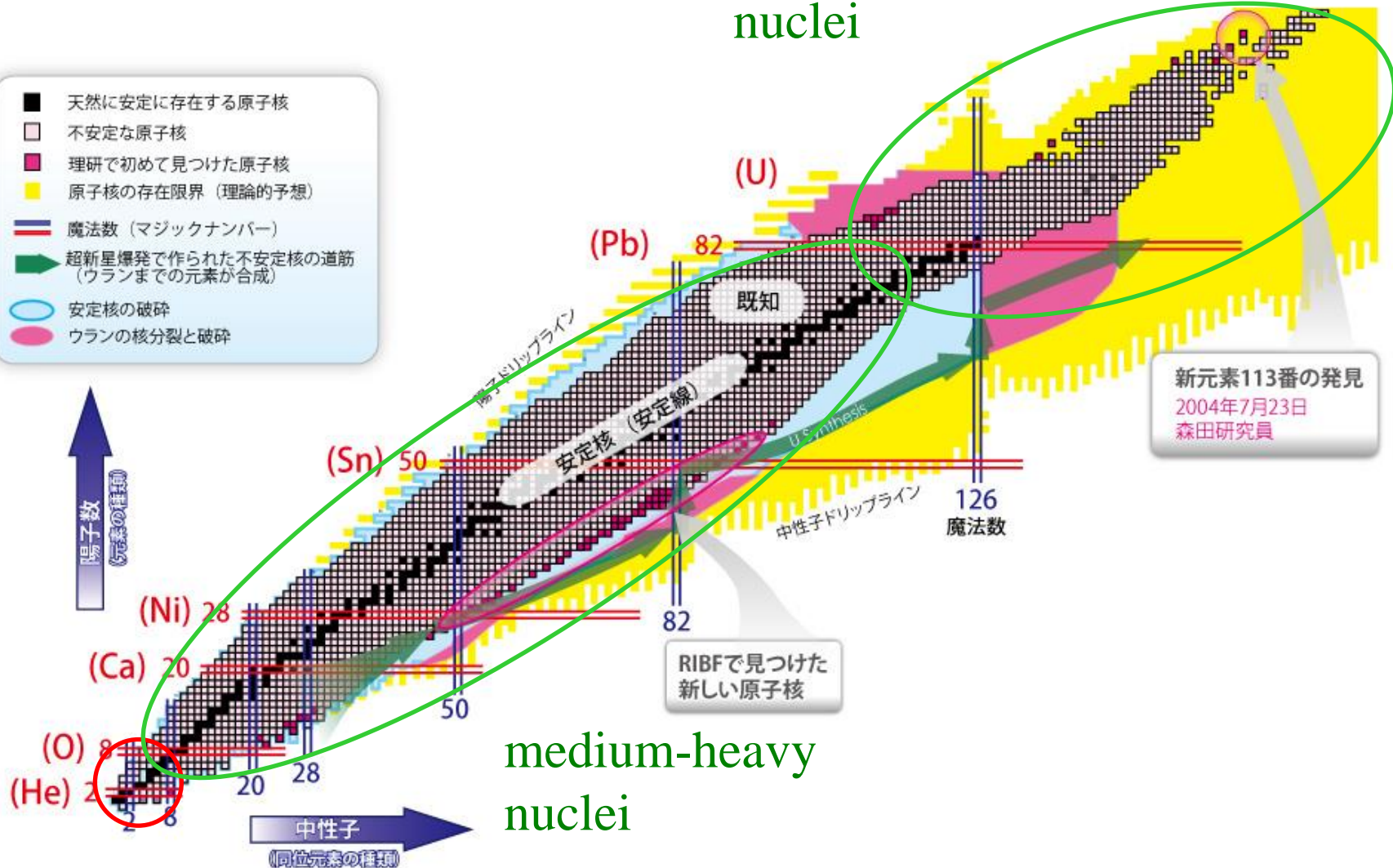


light nuclei

medium-heavy nuclei

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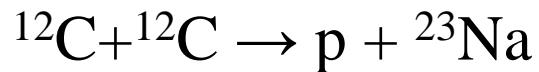
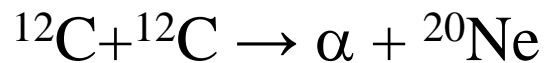
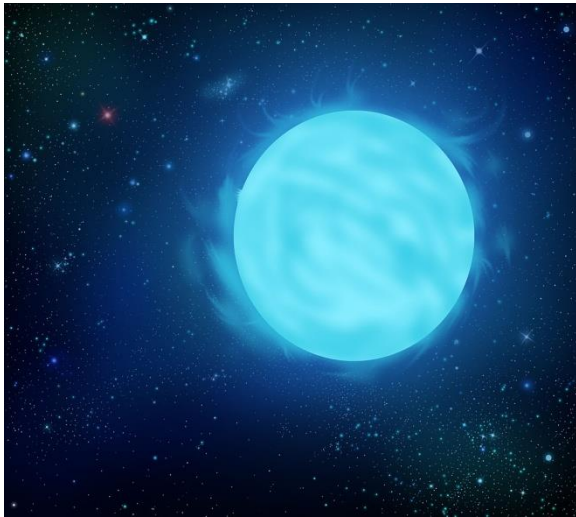
medium-heavy nuclei

➔ light nuclei

# Fusion of light nuclei: nuclear astrophysics

$^{12}\text{C}+^{12}\text{C}$  fusion : a key reaction in nuclear astrophysics

Carbon burning  
in massive stars



also

- ✓ Type Ia supernovae
- ✓ X-ray superburst

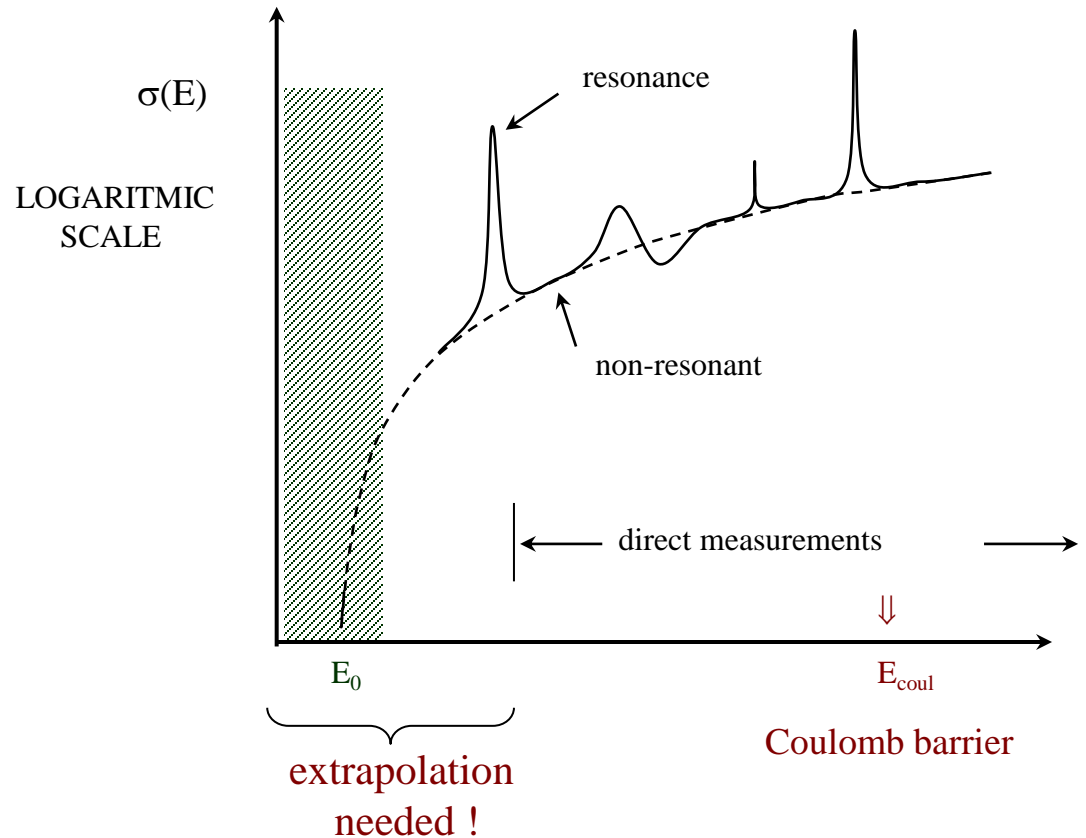
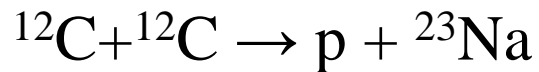
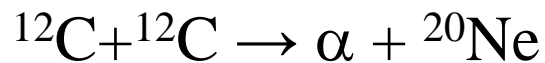
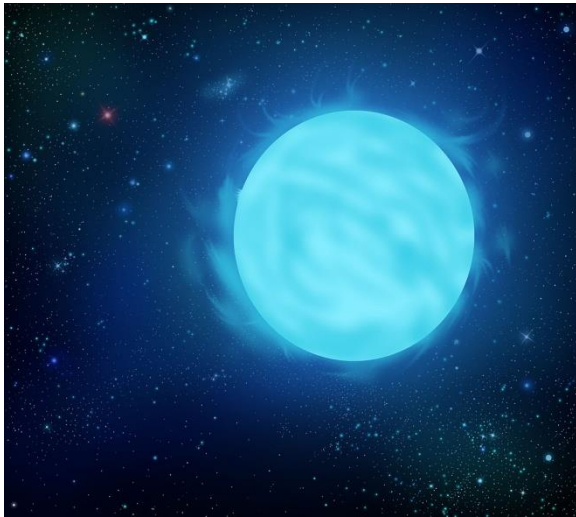


figure: M. Aliotta

# Fusion of light nuclei: nuclear astrophysics

$^{12}\text{C}+^{12}\text{C}$  fusion : a key reaction in nuclear astrophysics

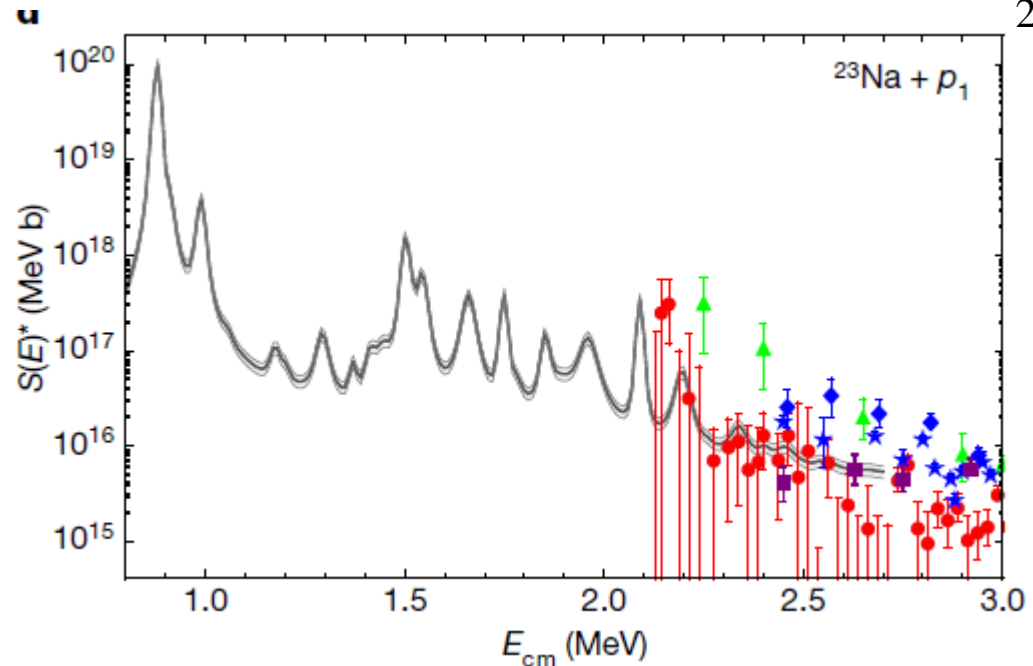
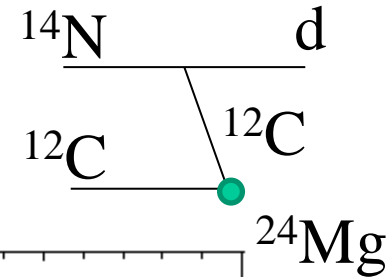
Carbon burning  
in massive stars



also

- ✓ Type Ia supernovae
- ✓ X-ray superburst

Trojan Horse Method

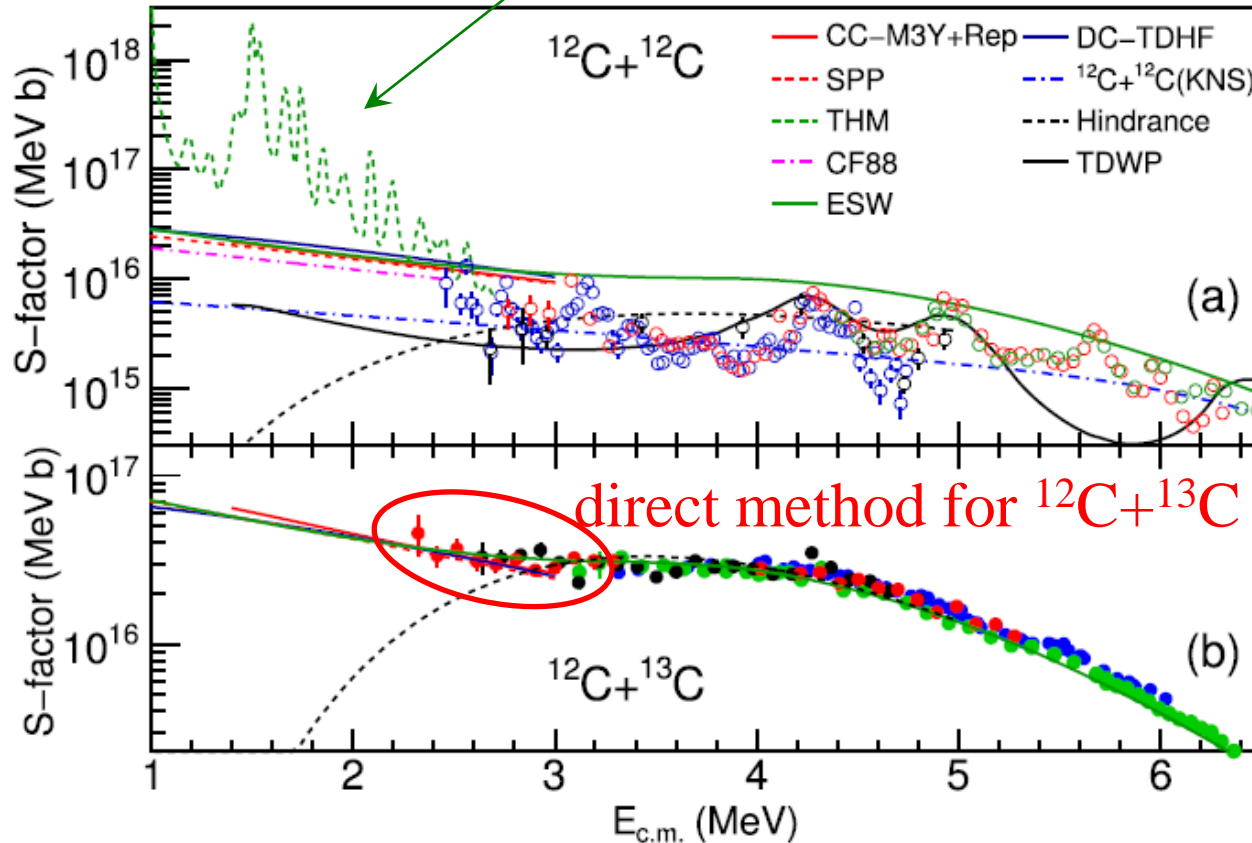


A. Tumino et al., Nature 557 ('18) 687

~ 25 times larger than before  
→ lots of debates

# $^{12}\text{C}+^{12}\text{C}$ fusion reaction

Trojan Horse Method



The THM data are not supported.

PWIA  $\rightarrow$  DWIA?

A.M. Mukhamedzhanov et al.,  
PRC99 ('19) 064618

cf. recent direct measurement for  $^{12}\text{C}+^{12}\text{C}$ :

W.P. Tan et al.,  
PRL124 ('20) 192702

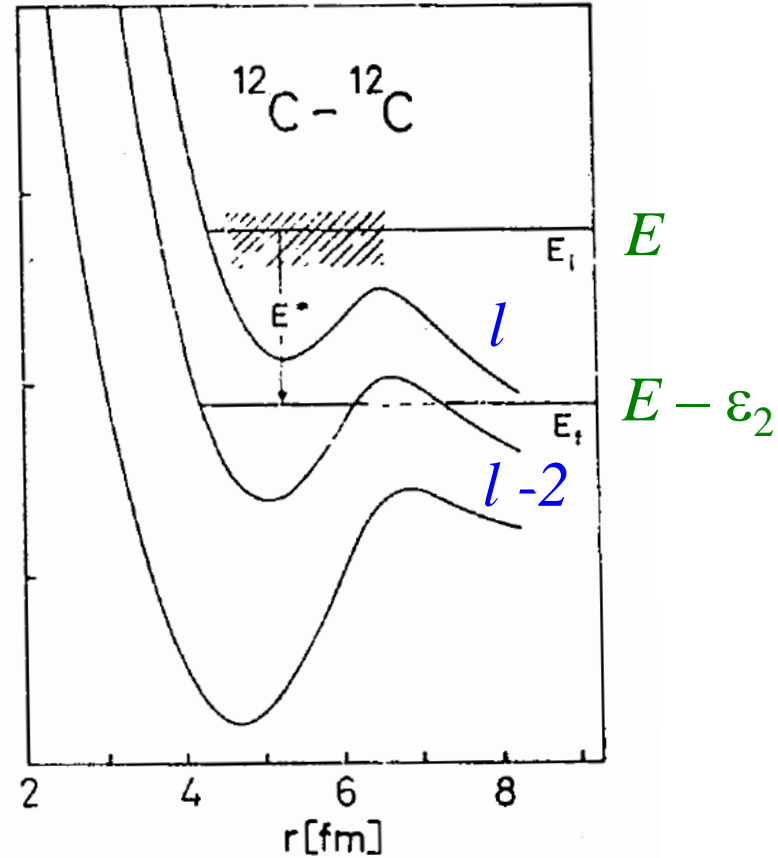
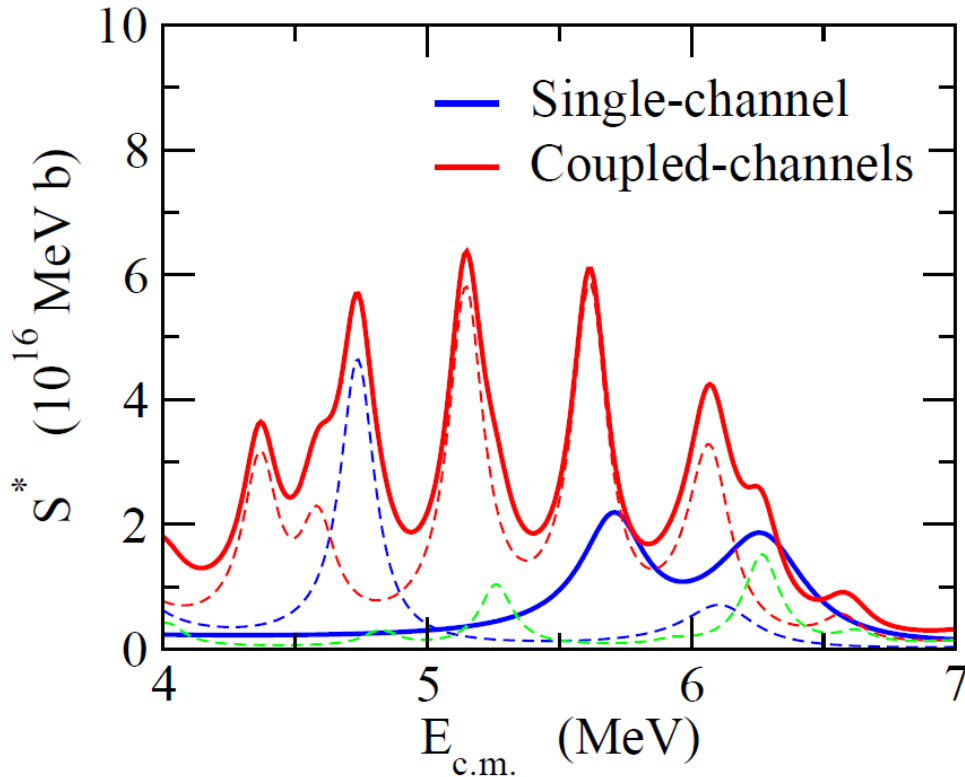
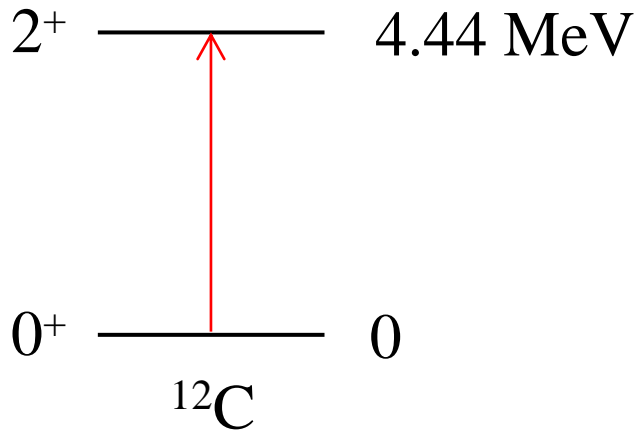


N.T. Zhang, ..., K.H., S. Kubono, ..., C.J. Lin, ...

XiaoDong Tang (IMP) et al., Phys. Lett. B801 (2020) 135170



# $^{12}\text{C}+^{12}\text{C}$ fusion: many resonance peaks

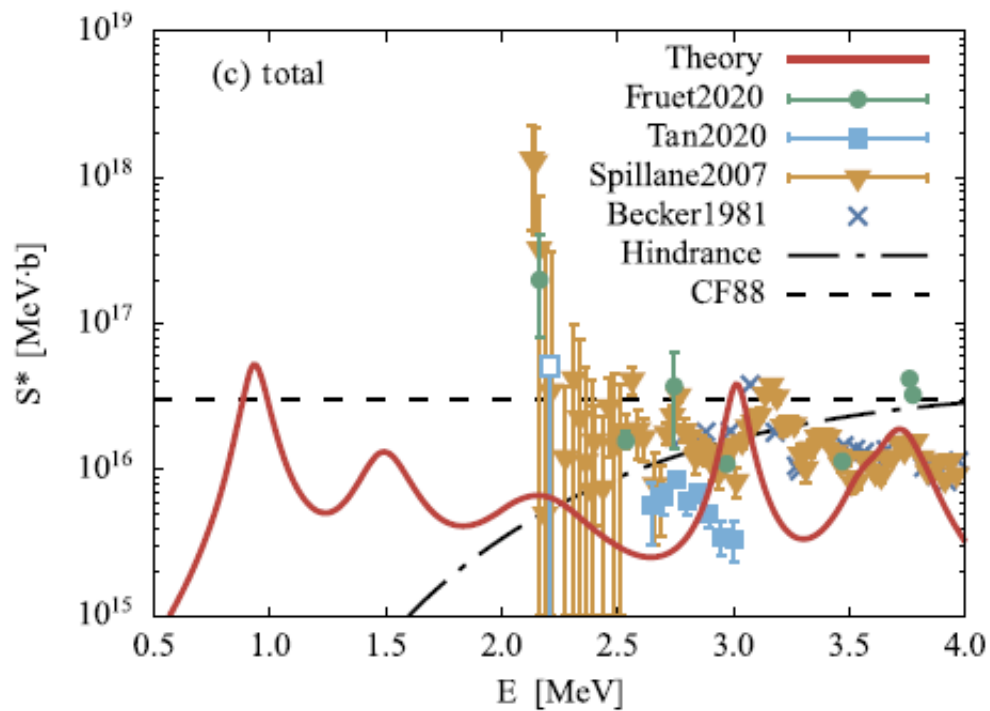
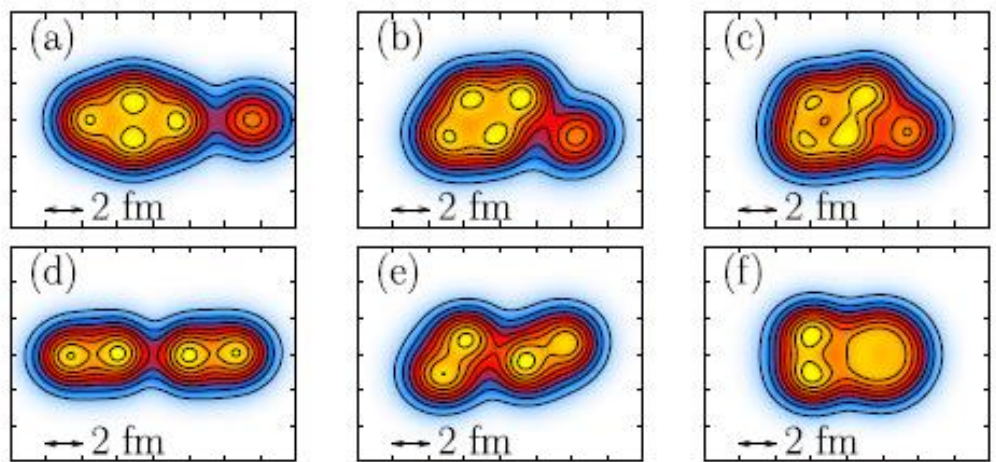


H.-J. Fink, W. Scheid,  
and W. Greiner,  
NPA188 ('72) 259

a kind of  
Feshbach resonance

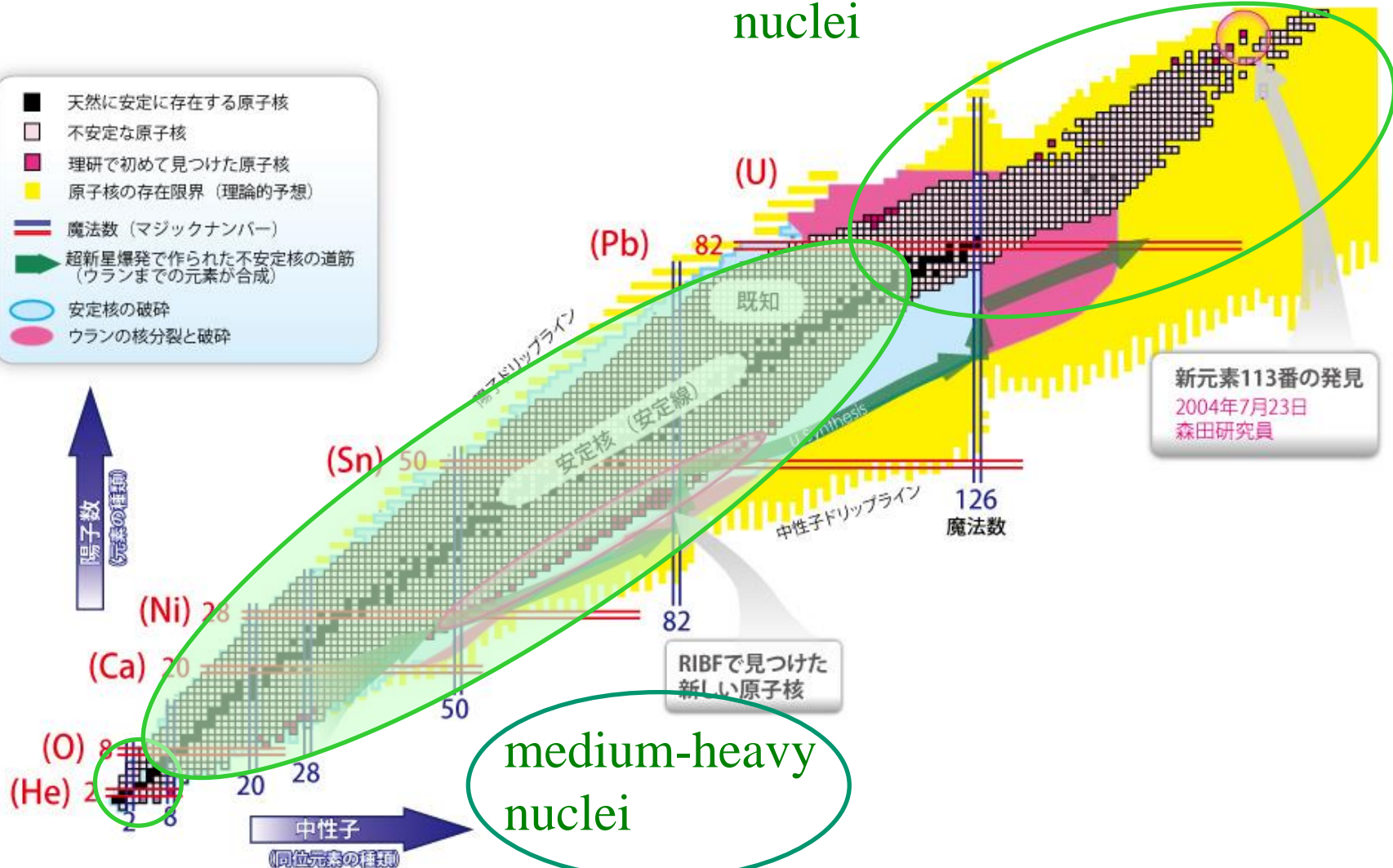
# A recent AMD calculation

Y. Taniguchi and M. Kimura, PLB823 ('21) 136790



# Heavy and Superheavy nuclei

- 天然に安定に存在する原子核
- 不安定な原子核
- 理研で初めて見つけた原子核
- 原子核の存在限界 (理論的予想)
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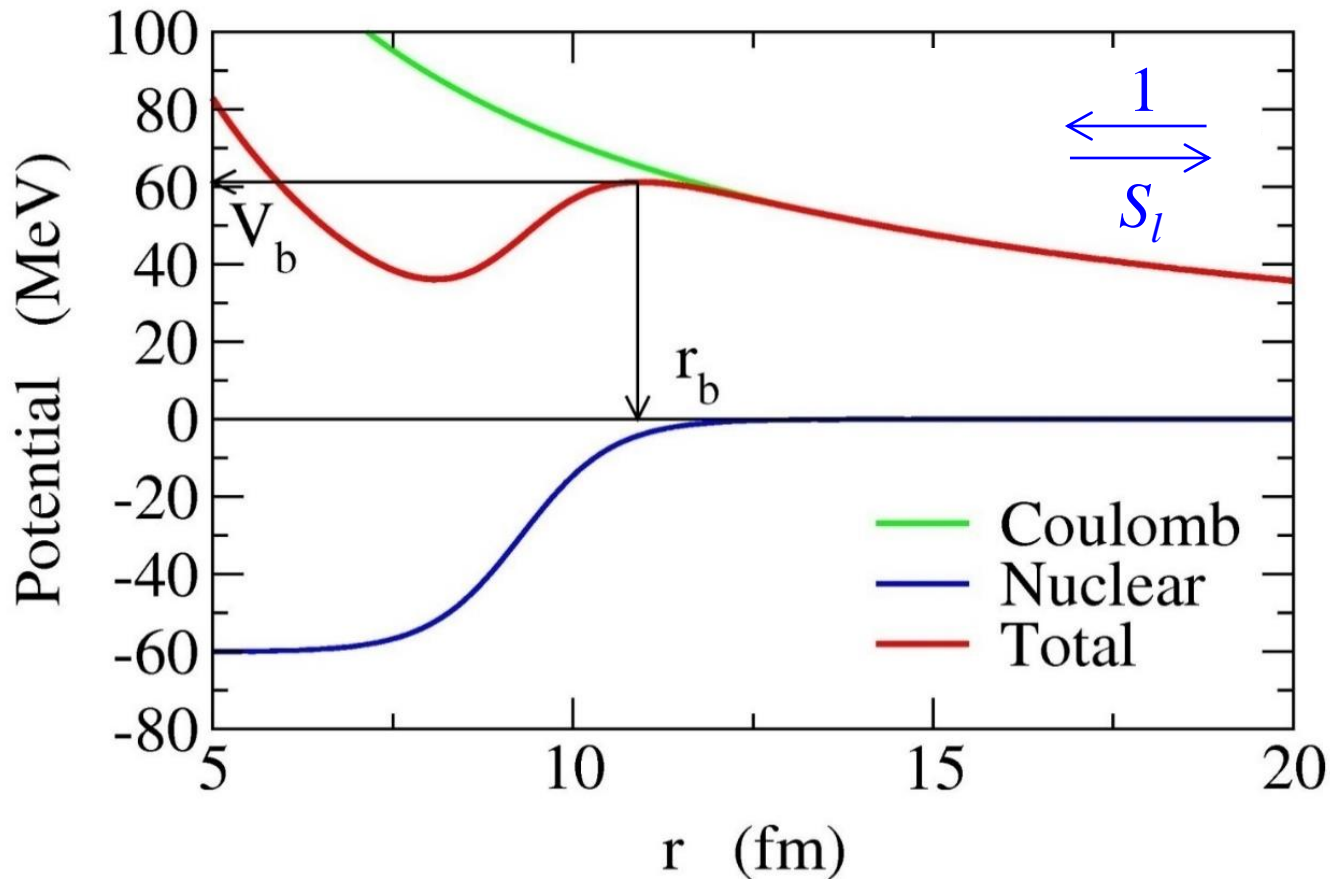
light nuclei

medium-heavy nuclei

# Fusion reactions of medium-heavy nuclei

potential model: inert nuclei (no structure)

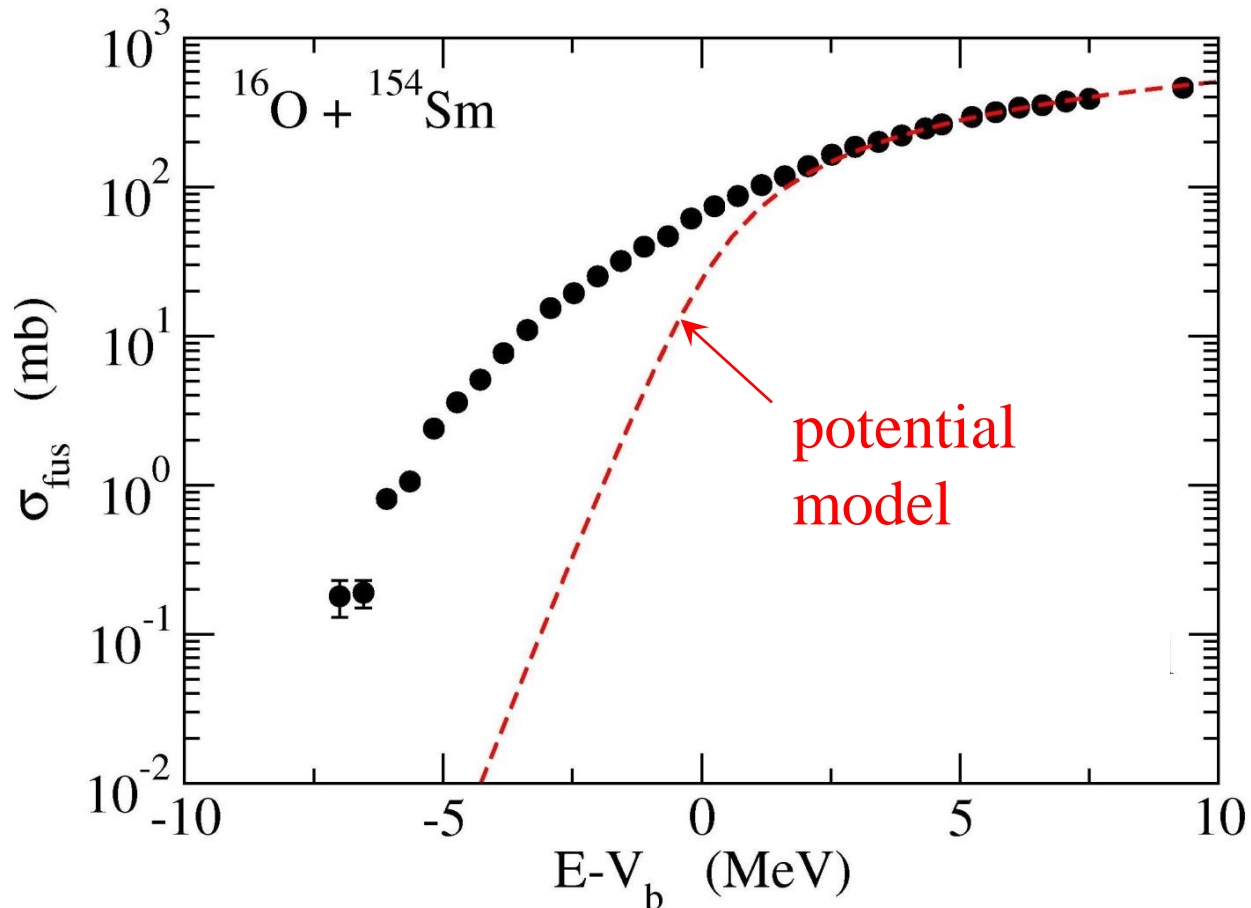
$$\sigma_{\text{fus}} = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - |S_l|^2)$$



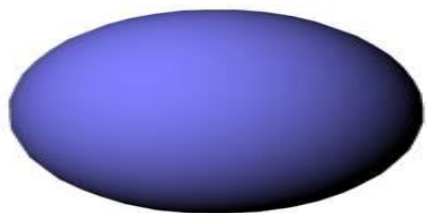
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# $^{154}\text{Sm}$ : a typical deformed nucleus



$^{154}\text{Sm}$

(MeV)

0.903 —————  $8^+$

0.544 —————  $6^+$

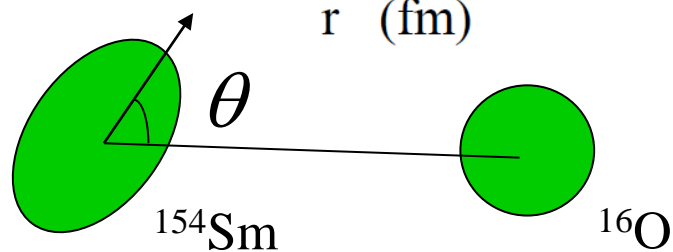
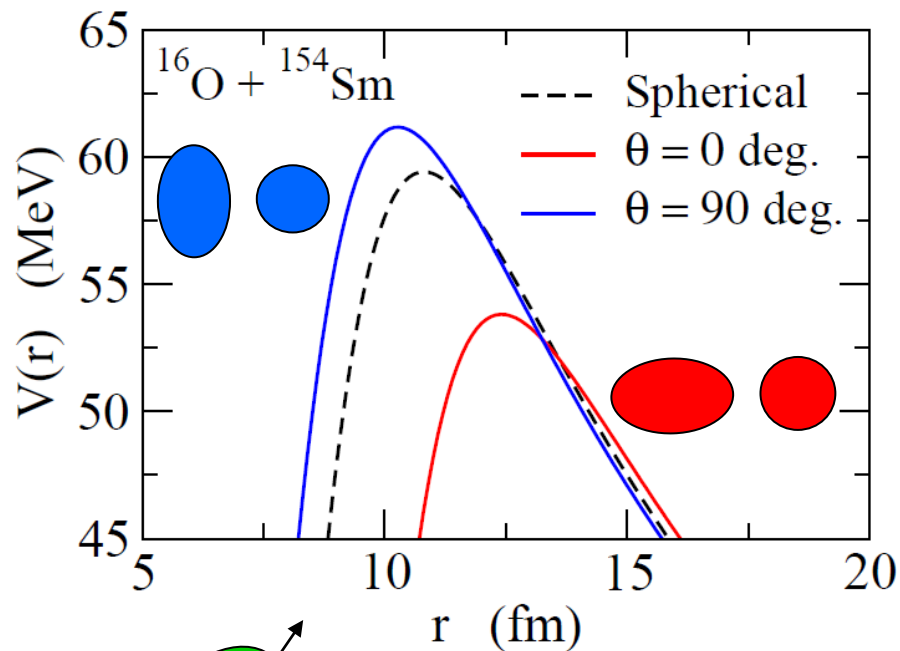
0.267 —————  $4^+$

0.082 —————  $2^+$

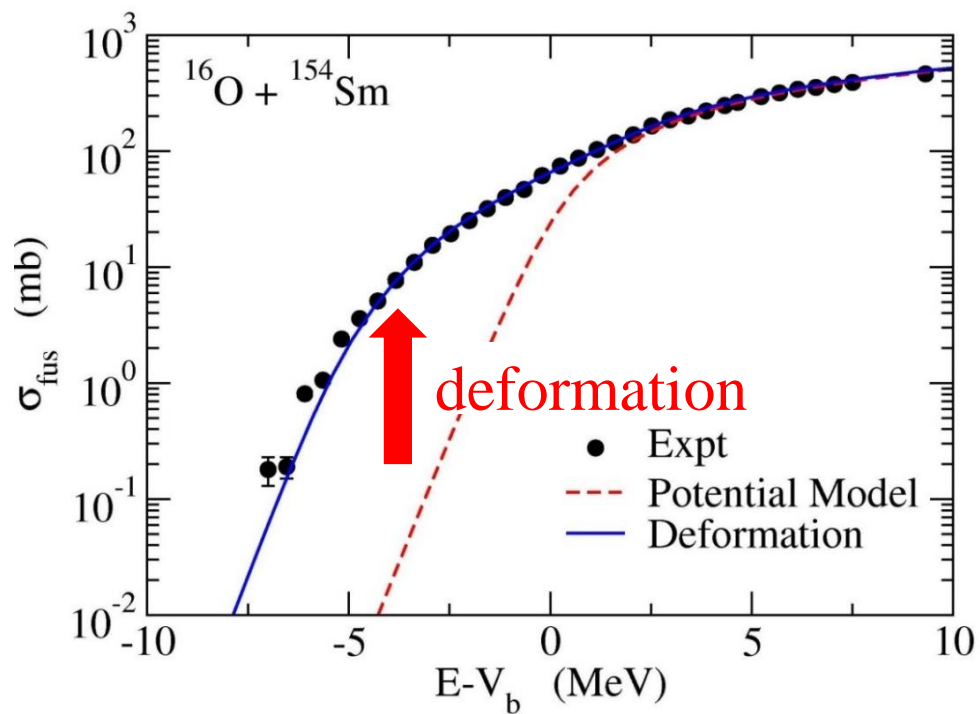
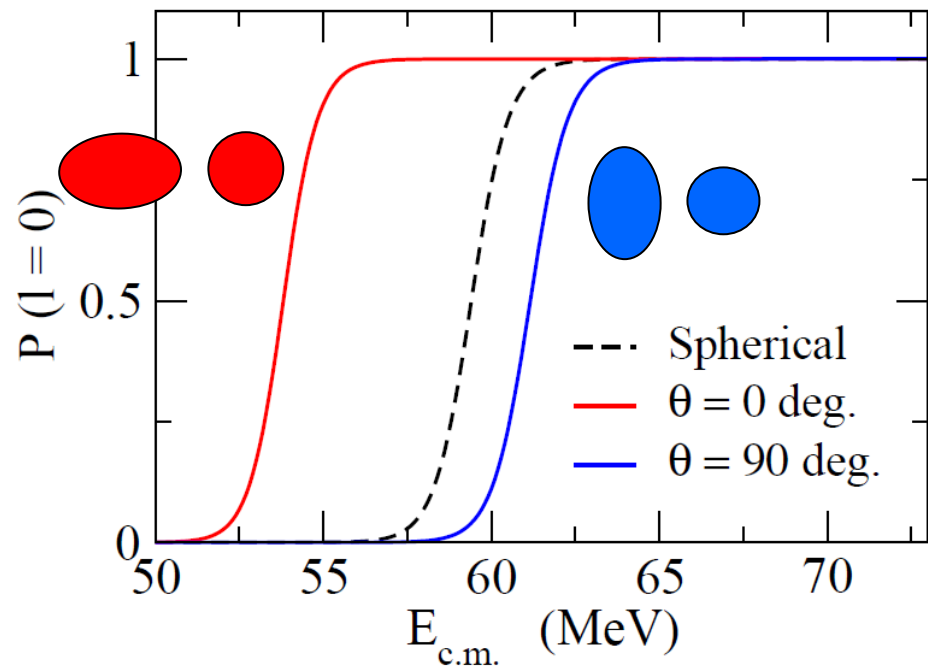
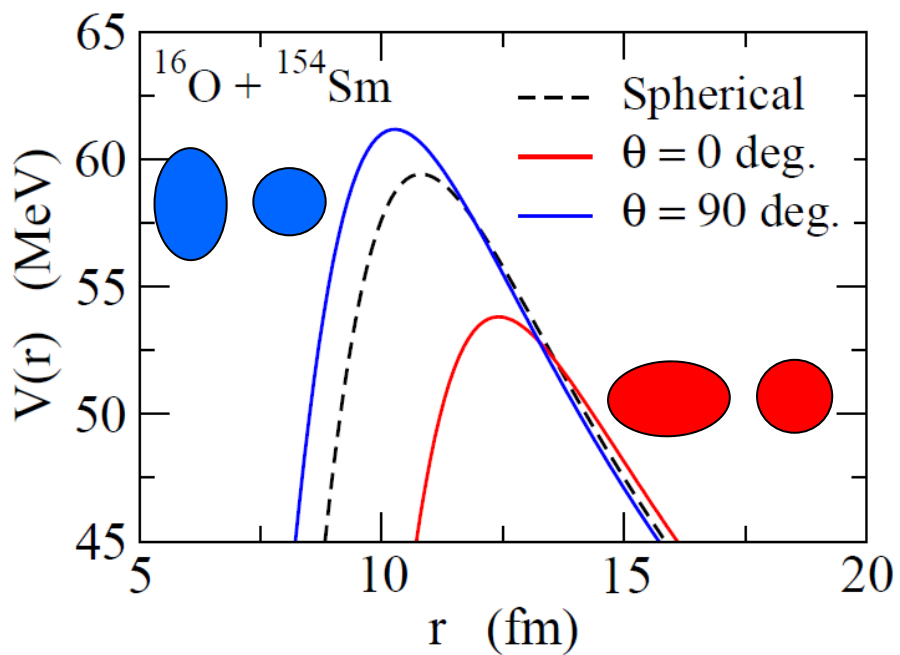
0 —————  $0^+$

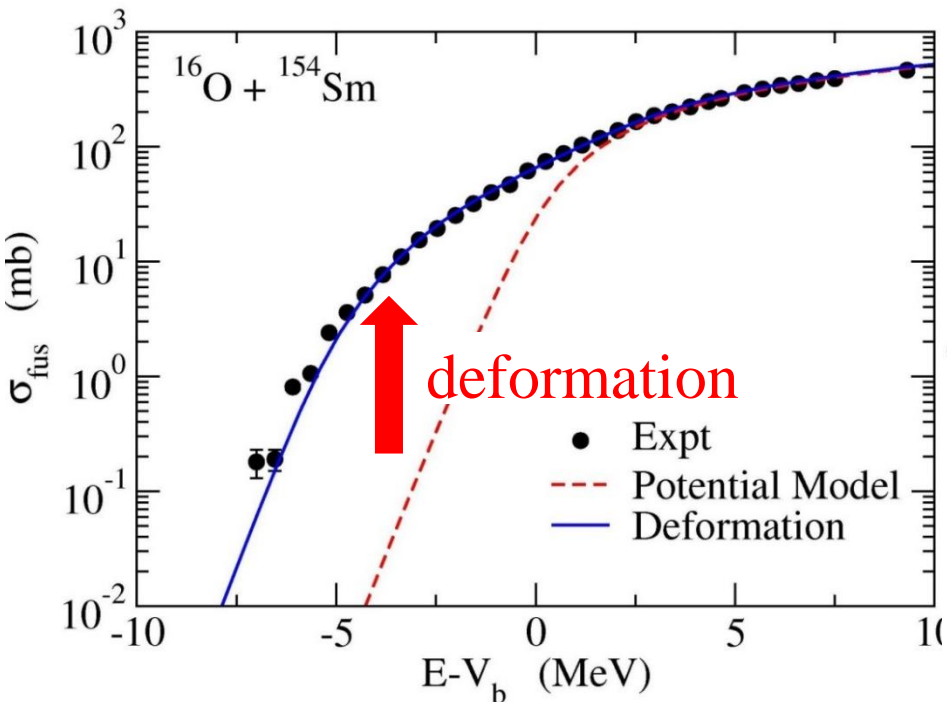
$^{154}\text{Sm}$

rotational spectrum

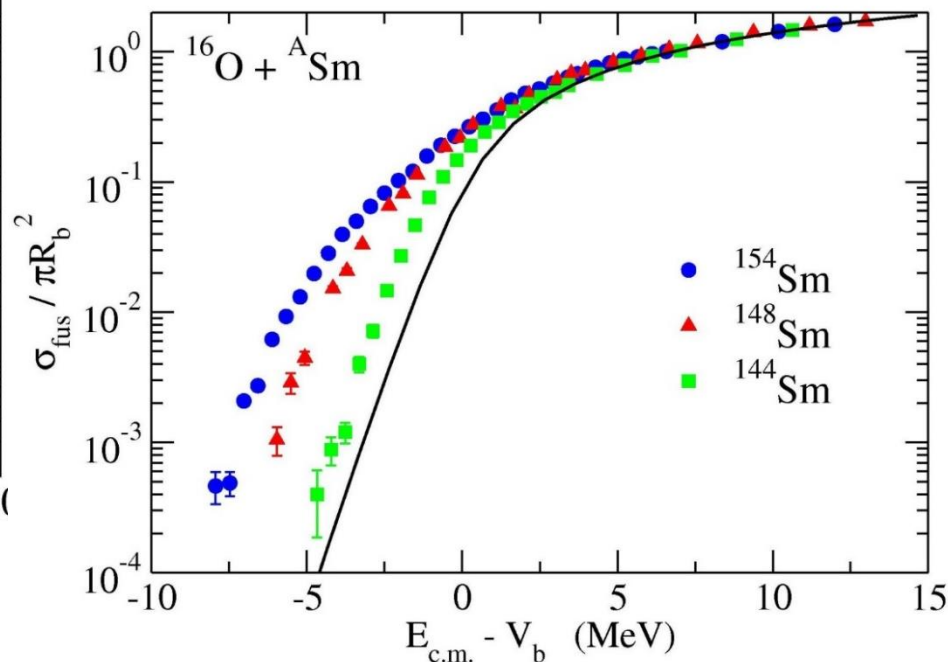


$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

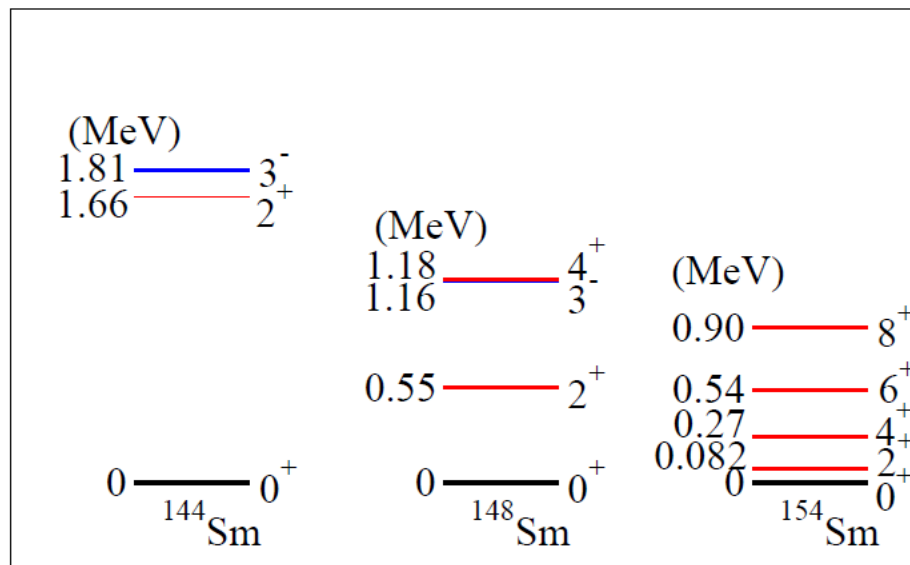




similar enhancement  
for non-deformed nuclei



strong correlation  
with nuclear spectrum  
→ coupling assisted  
tunneling phenomena



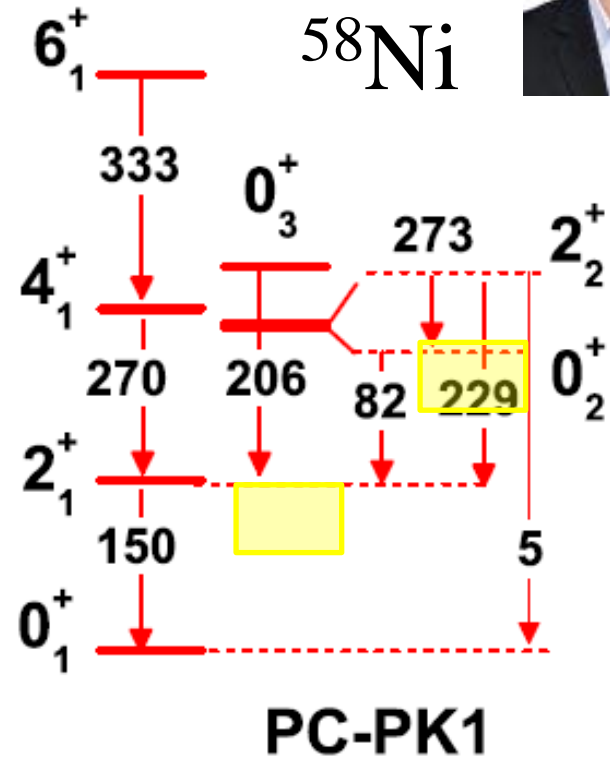
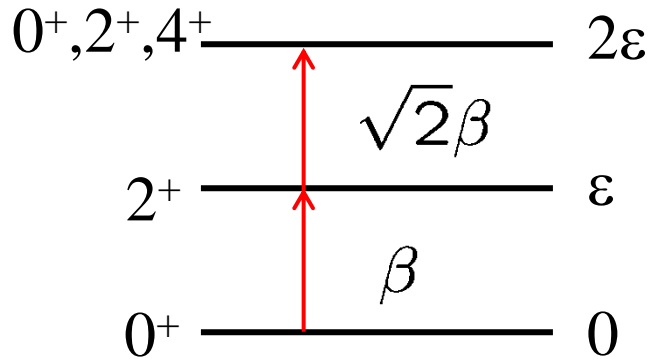


# Semi-microscopic modelling of subbarrier fusion reactions

K.H. and J.M. Yao, PRC91('15) 064606

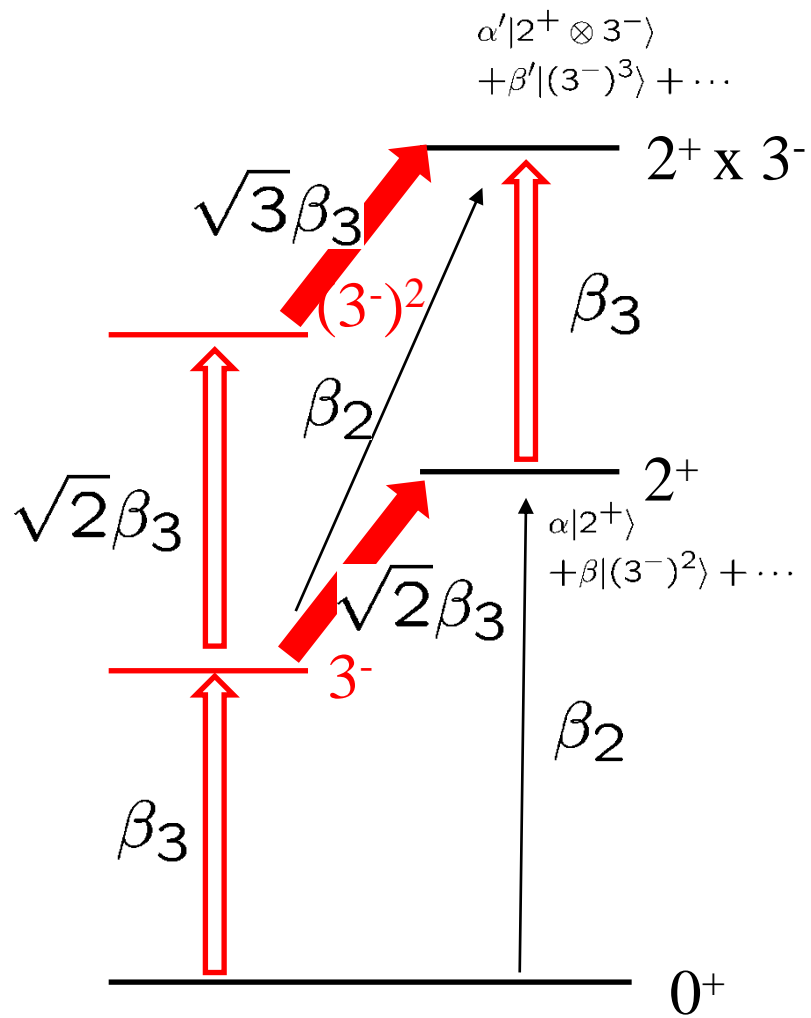
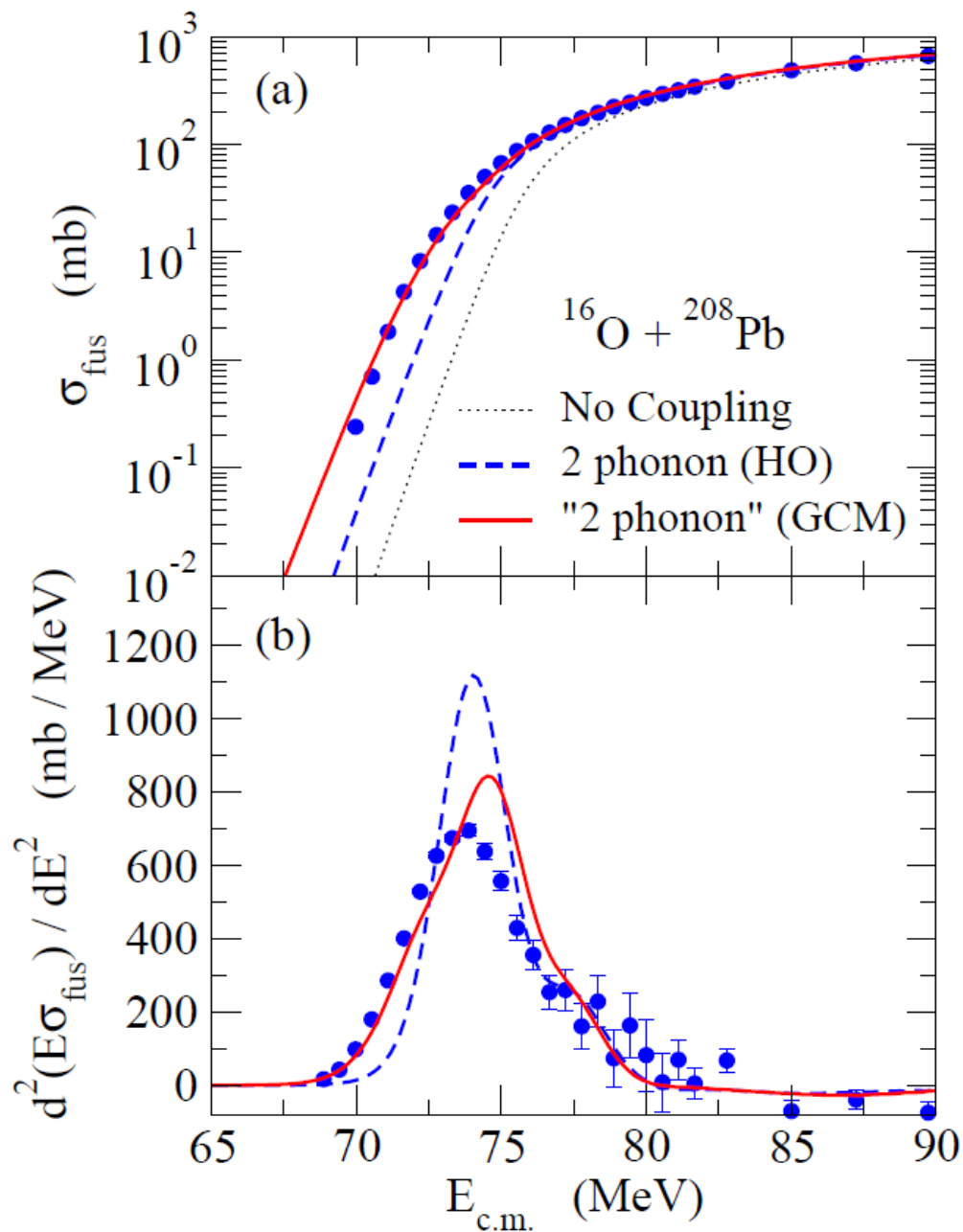


simple harmonic oscillator



Beyond-mean-field method  
anharmonicity of phonon spectra

→ C.C. calculations with  
a phenomenological potential



J.M. Yao and K.H.,  
PRC94 ('16) 11303(R)

# From phenomenological approach to microscopic approach

Macroscopic (phenomenological)

C.C. with collective model

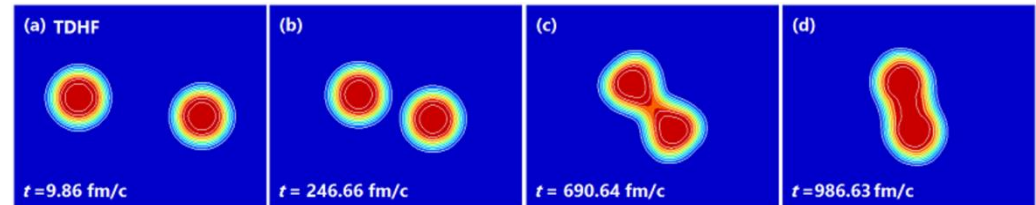
C.C. with inputs from  
microscopic nuclear  
structure calculations

C.C. with inputs based  
on TDHF

TDHF simulations

Microscopic

TDHF = Time Dependent Hartree-Fock

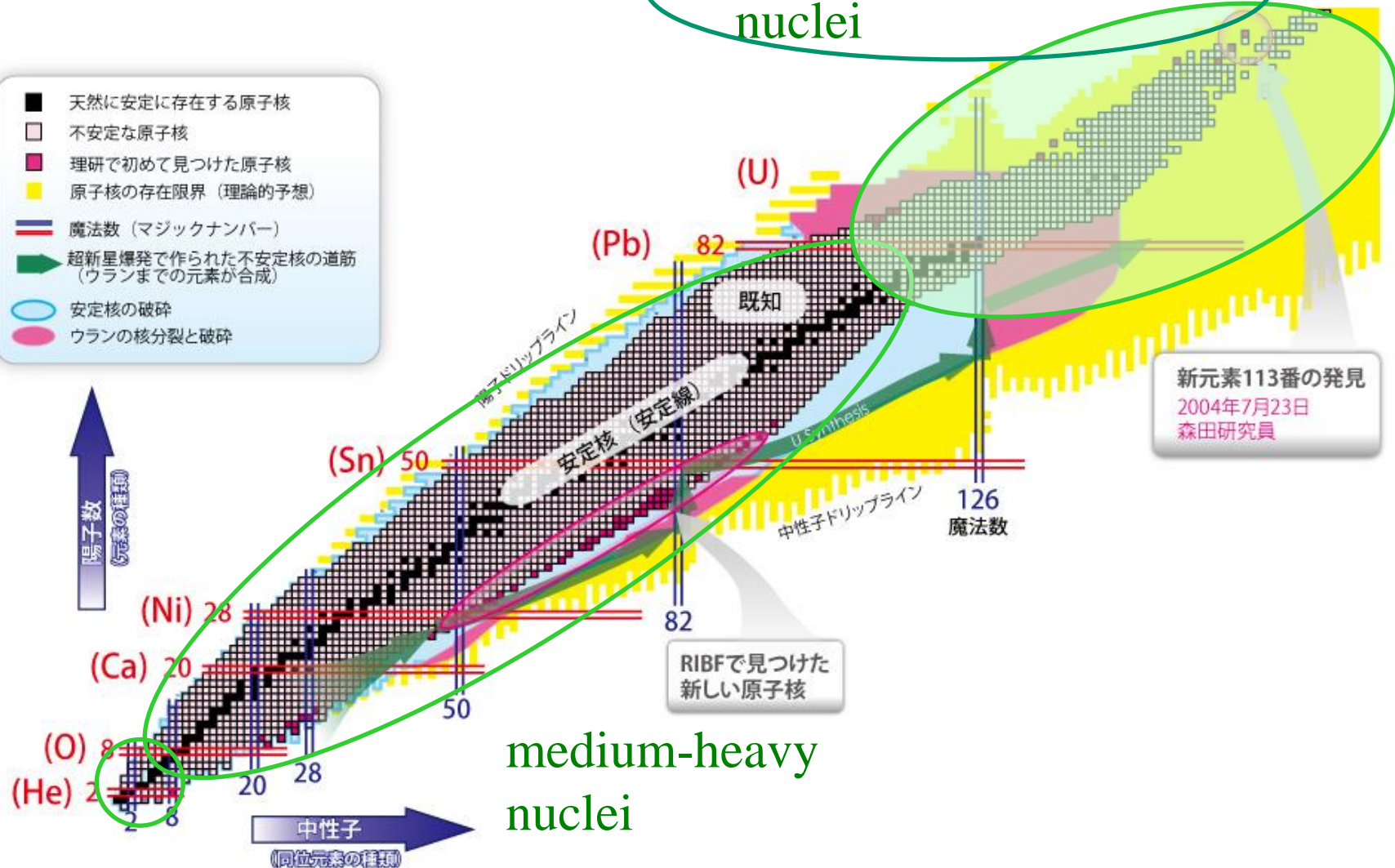


S. Ebata, T. Nakatsukasa, JPC Conf. Proc. 6 ('15)

ab initio, but no tunneling

# Heavy and Superheavy nuclei

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light nuclei

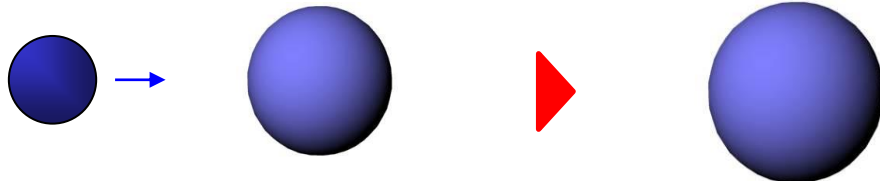
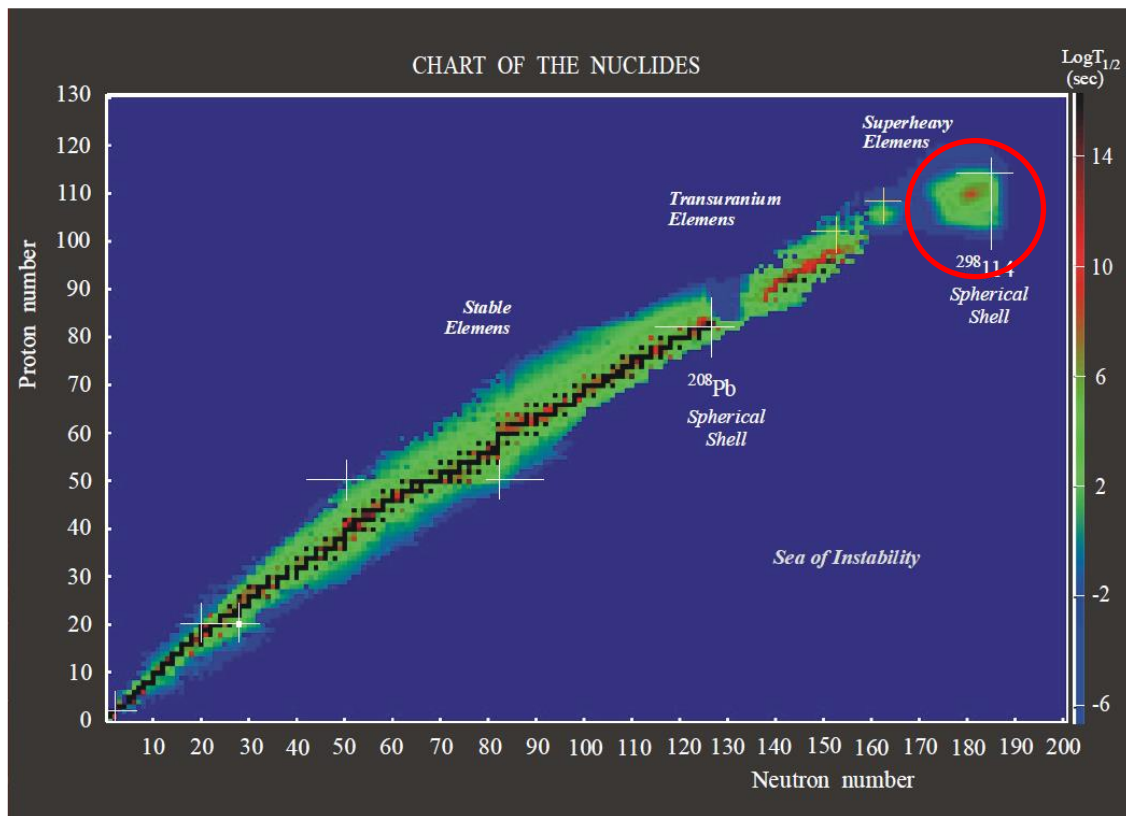
medium-heavy nuclei

# Superheavy elements

the island of stability (安定的島)

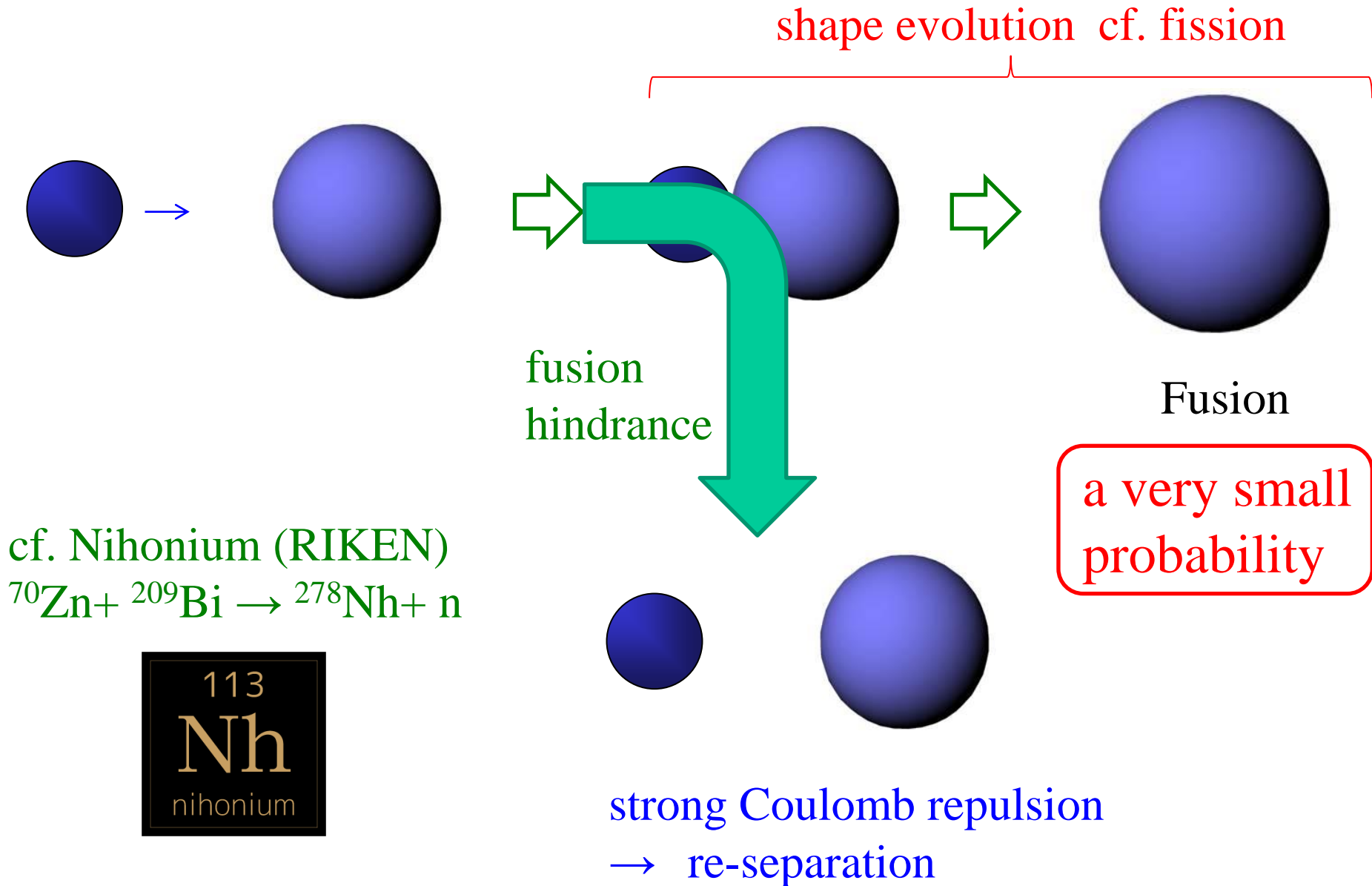


November, 2016

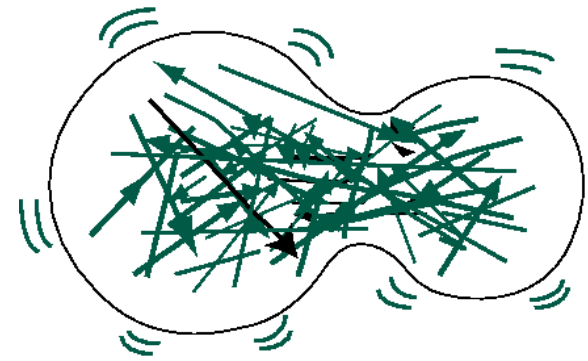
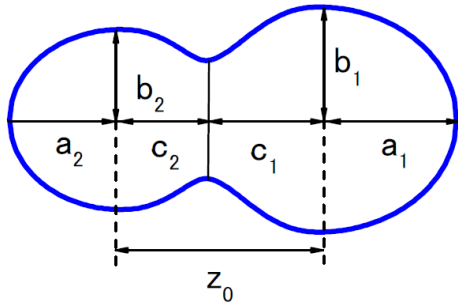


Heavy-ion fusion reaction

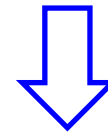
# Fusion of heavy nuclei and superheavy elements



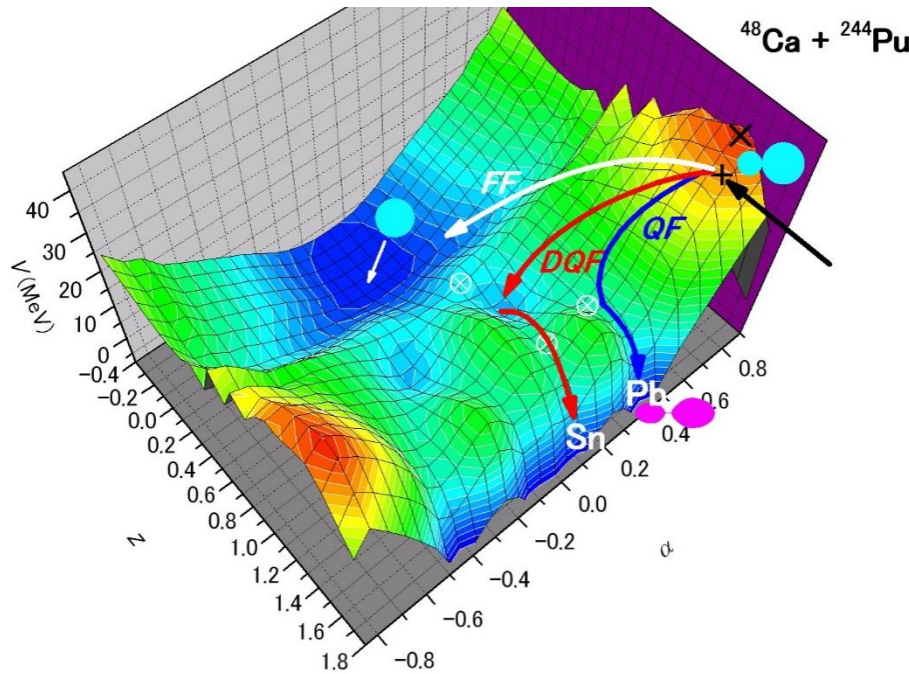
# Nuclear shape evolution



nucleus = many-body system  
of nucleons



nuclear intrinsic d.o.f.  
: internal environment  
→ physics of open quantum  
systems



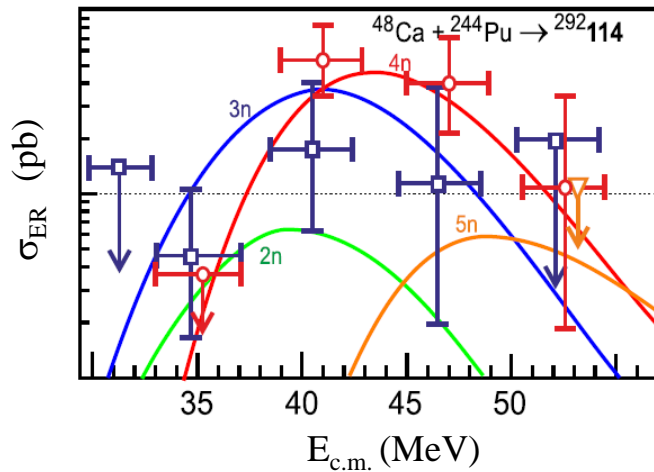
cf. Classical Langevin equation

$$m \frac{d^2 q}{dt^2} = - \frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$

Y. Aritomo, K. Hagino, K. Nishio,  
and S. Chiba, PRC85 (2012) 044614

# Nuclear shape evolution

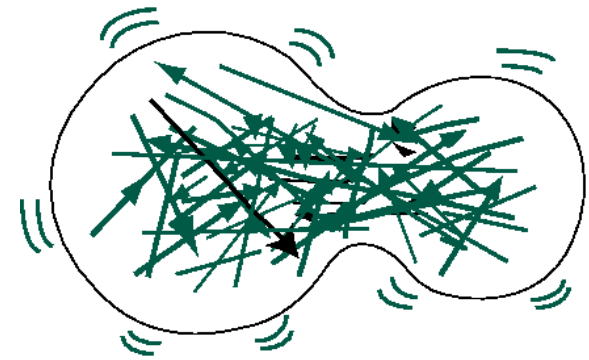
- successful as a phenomenological approach



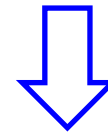
V.I. Zagrebaev and W. Greiner (2015)



- a more microscopic approach?
- quantum effects?



nucleus = many-body system  
of nucleons



nuclear intrinsic d.o.f.

: internal environment

→ physics of open quantum  
systems

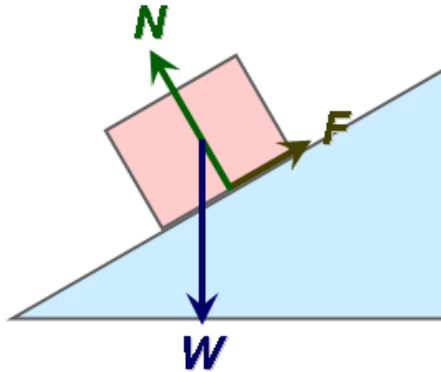
cf. Classical Langevin equation

$$m \frac{d^2 q}{dt^2} = - \frac{dV(q)}{dq} - \gamma \frac{dq}{dt} + R(t)$$



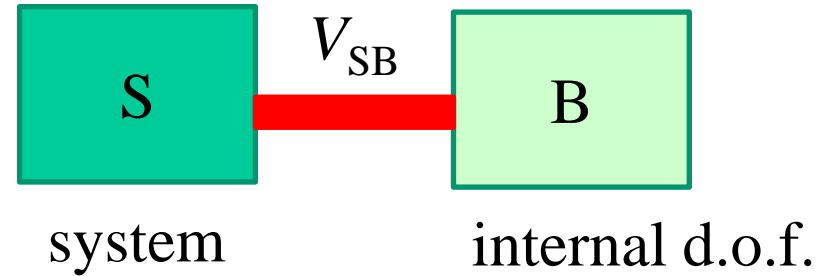
# Fusion from a viewpoint of open quantum systems

classical mechanics



heat generation when a rigid body stops

quantum mechanics

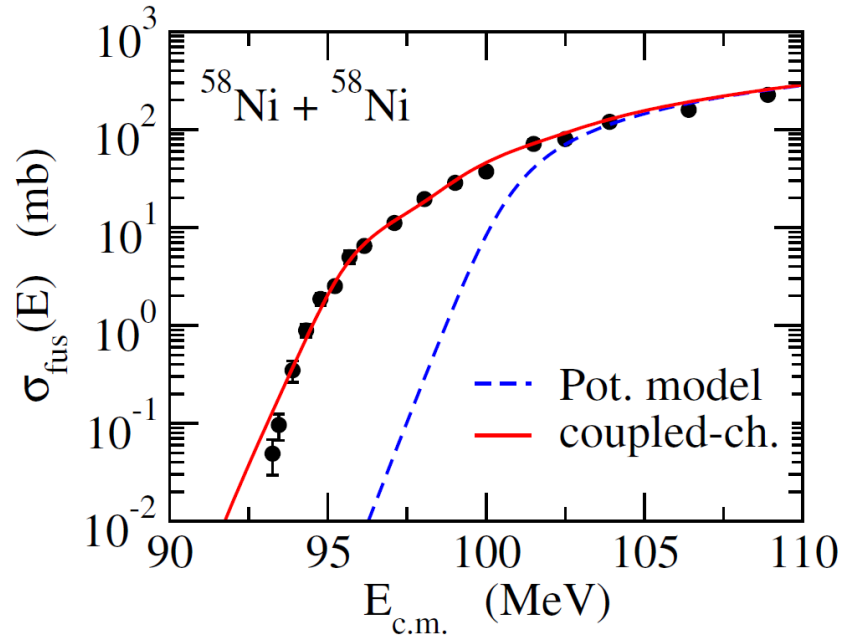


Caldeira-Leggett model

$$H_S = \frac{p^2}{2m} + V(q)$$
$$H_{\text{int}} = \sum_i \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 x_i^2$$

a collection of H.O.

# Fusion from a viewpoint of open quantum systems



cf. a vib. coupling in subbarrier fusion

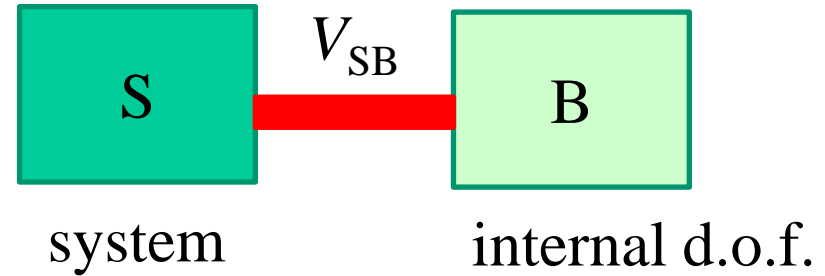
2.90 MeV ——— 0<sup>+</sup>, 2<sup>+</sup>, 4<sup>+</sup>

1.45 MeV ——— 2<sup>+</sup>

————— 0<sup>+</sup>

<sup>58</sup>Ni

quantum mechanics



Caldeira-Leggett model

$$H_S = \frac{p^2}{2m} + V(q)$$

$$H_{\text{int}} = \sum_i \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 x_i^2$$

a collection of H.O.

→ C.C. calculations

# Fusion from a viewpoint of open quantum systems

## Caldeira-Leggett model

$$H_S = \frac{p^2}{2m} + V(q)$$
$$H_{\text{int}} = \sum_{i=1}^{\infty} (a_i^\dagger a_i + 1/2) \hbar \omega_i$$

## cf. a “two-phonon” state

$$2.90 \text{ MeV} \quad \equiv \equiv \quad 0^+, 2^+, 4^+$$

$$1.45 \text{ MeV} \quad \text{---} \quad 2^+$$

$$\text{---} \quad 0^+$$

$^{58}\text{Ni}$

$$|2ph\rangle = \sum_I \langle 2020 | I0 \rangle |\phi_I\rangle$$

how to deal with a huge number of phonon modes?

→ an efficient truncation scheme

$$b_k^\dagger = \sum_{i=1}^{\infty} C_{ki} a_i^\dagger \quad (k = 1, \dots, K)$$

$$e^{-i\omega t} \sim \sum_{k=0}^K \eta_k(\omega) J_k(t)$$

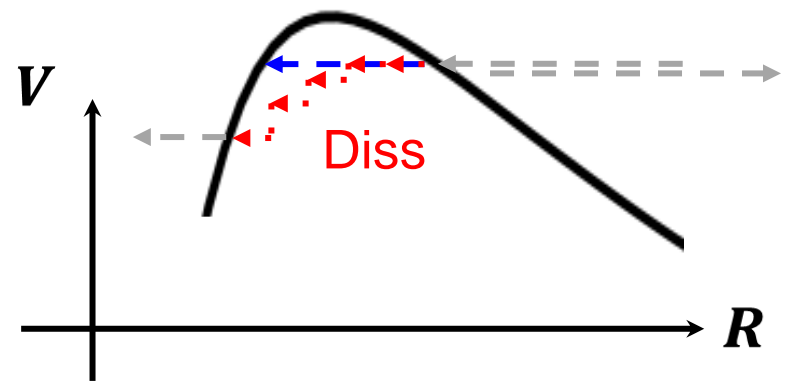
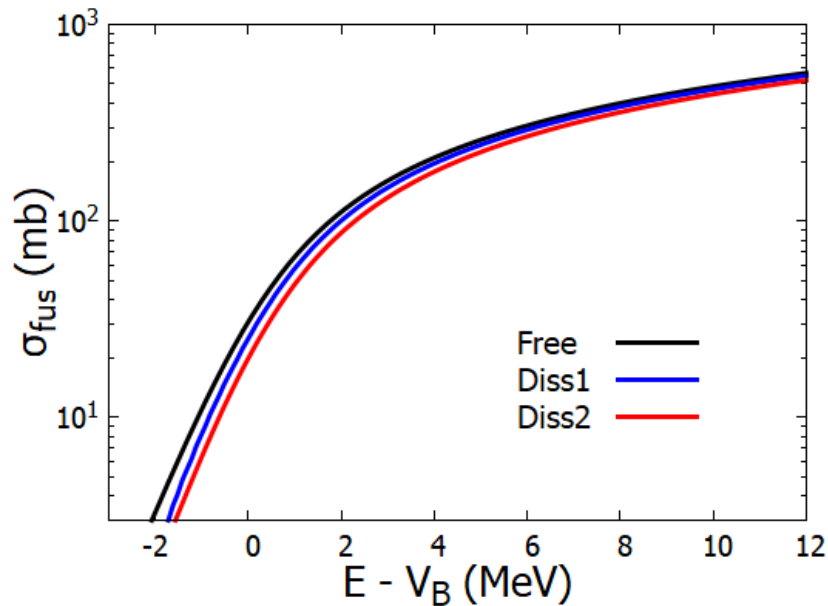
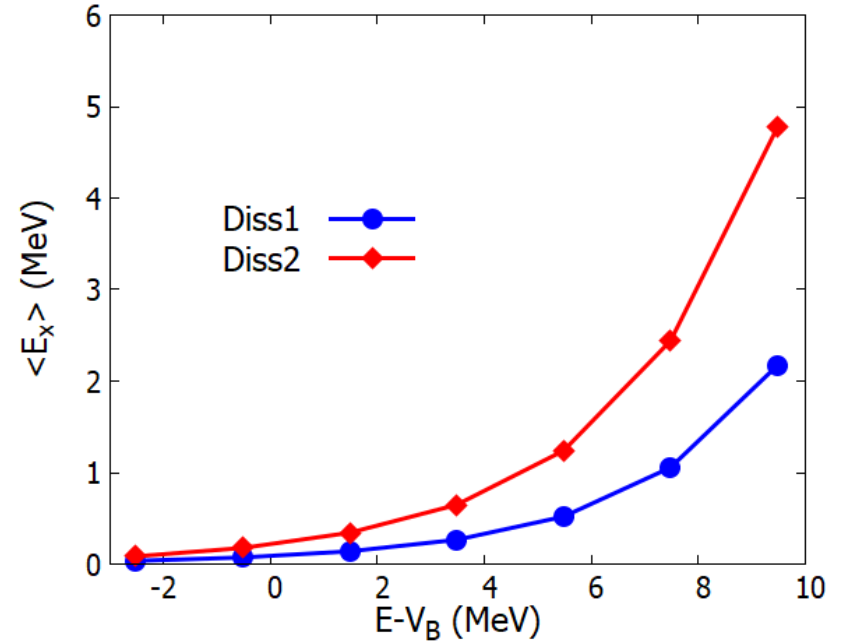
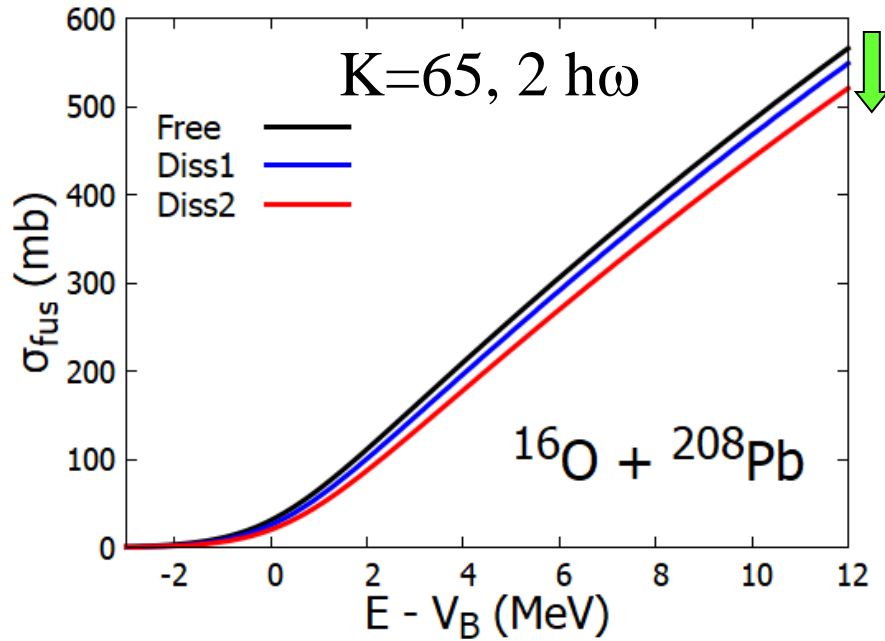


$$\rightarrow b_k^\dagger = \sum_i \left[ \frac{d_i}{\hbar} \eta_k(\omega_i) \right] a_i^\dagger$$

M. Tokieda and K. Hagino,  
Ann. of Phys. 412 (2020) 168005  
Front. in Phys. 8 (2020) 8.

# fusion cross sections

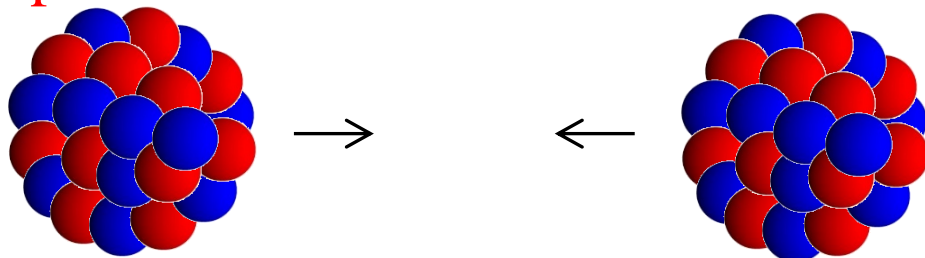
$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1)(1 - R_l(E))$$



M. Tokieda, Ph.D. thesis (2021),  
Tohoku University

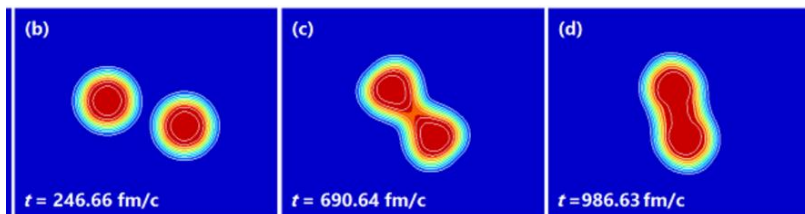
# Towards a microscopic nuclear reaction theory

many-body problems



still very challenging

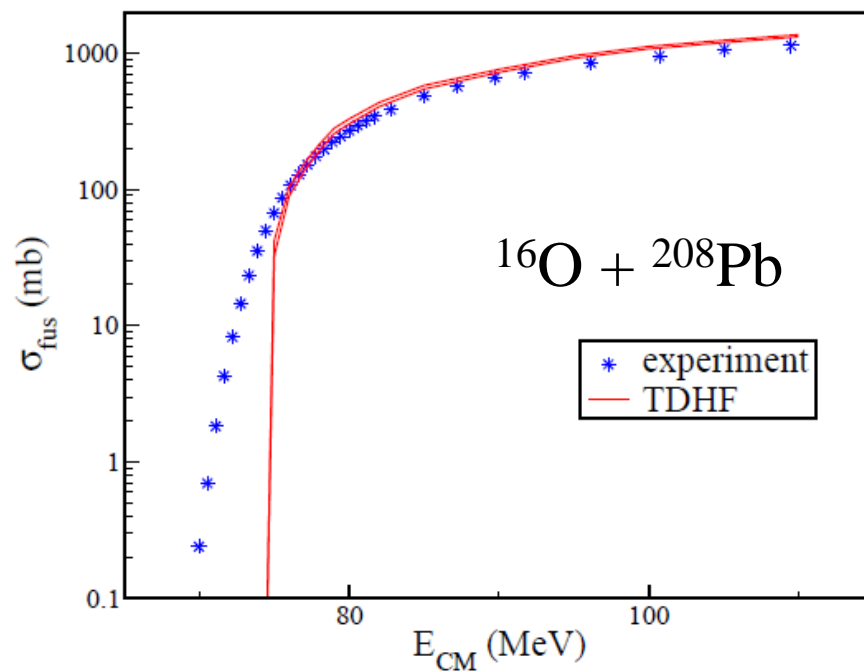
Time-dependent mean-field theory (TDHF/TDDFT)



S. Ebata, T. Nakatsukasa,  
JPC Conf. Proc. 6 ('15) 020056

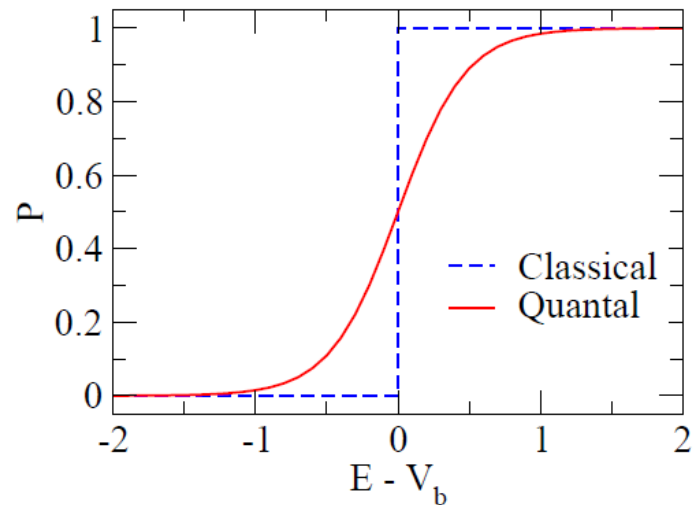
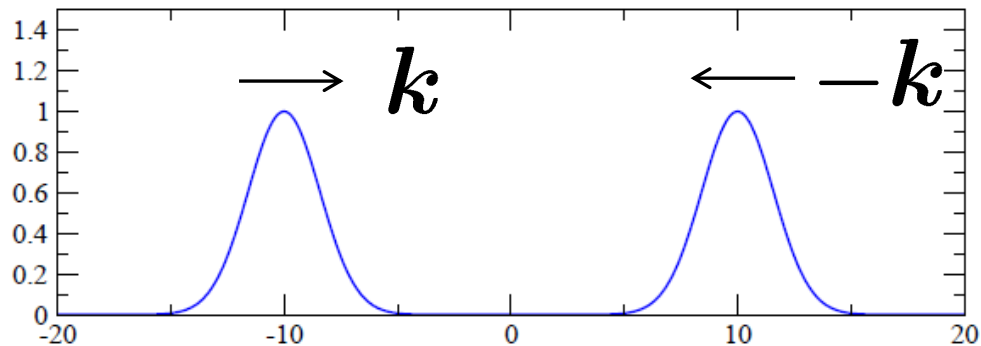
(semi) classical  $\rightarrow$  no tunneling

a microscopic understanding of  
many-body tunneling?



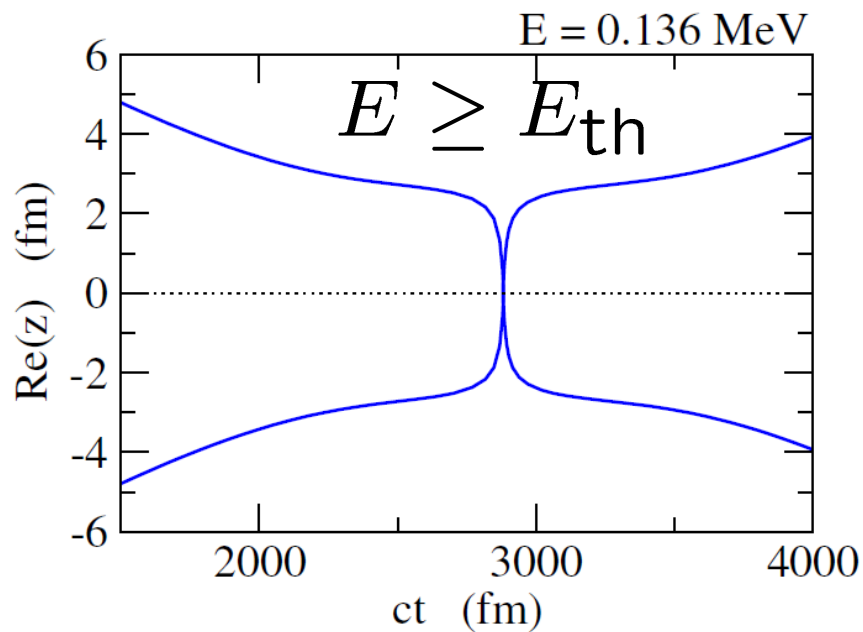
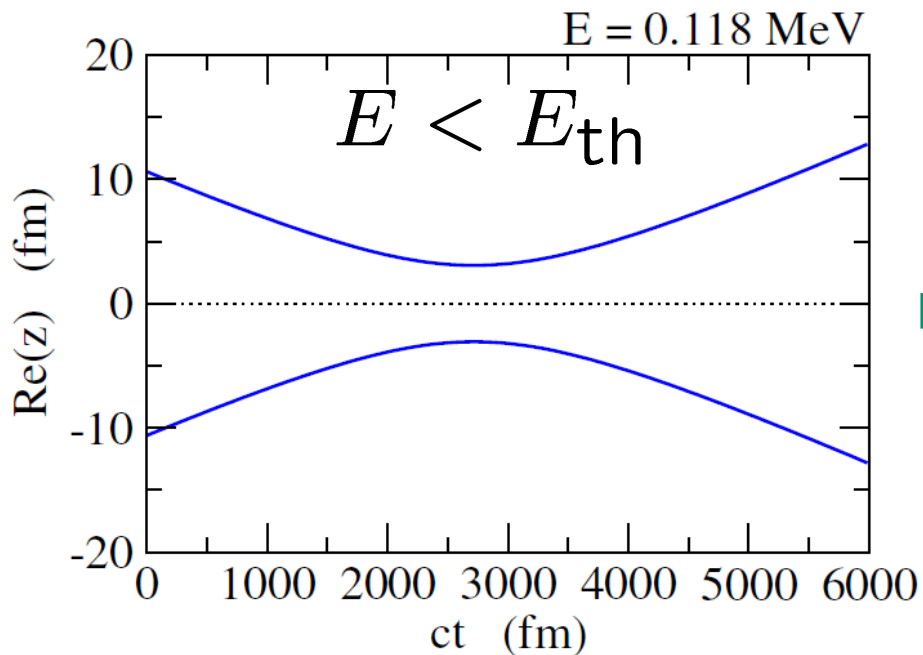
# TDHF

$$\Psi(t) = \Phi_{SD}(t)$$

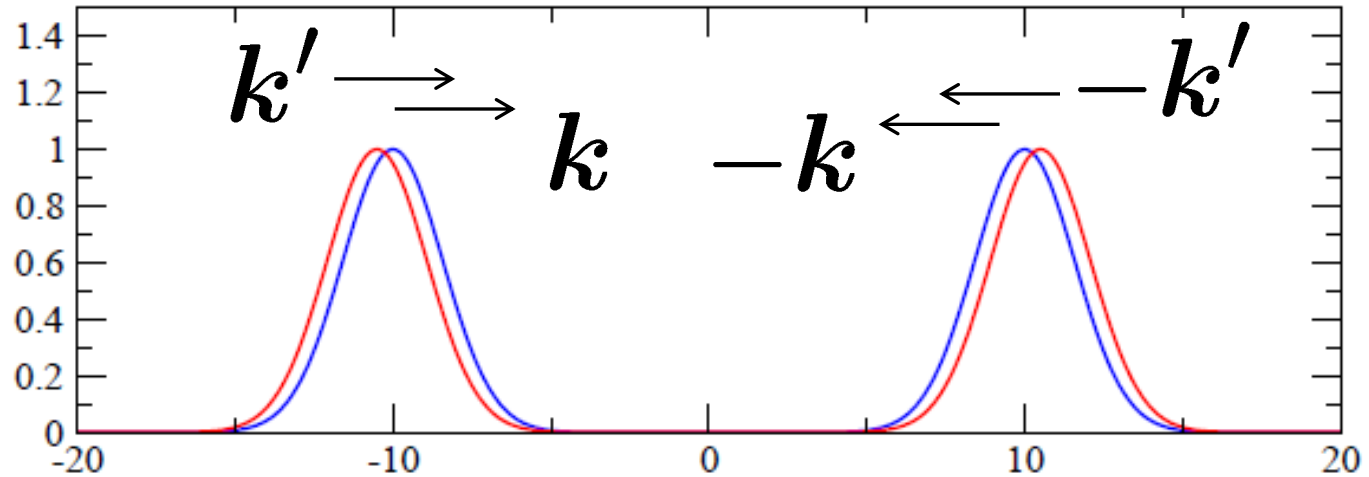


a single Slater determinat for a many-body wave function

$\alpha + \alpha$  in 1D



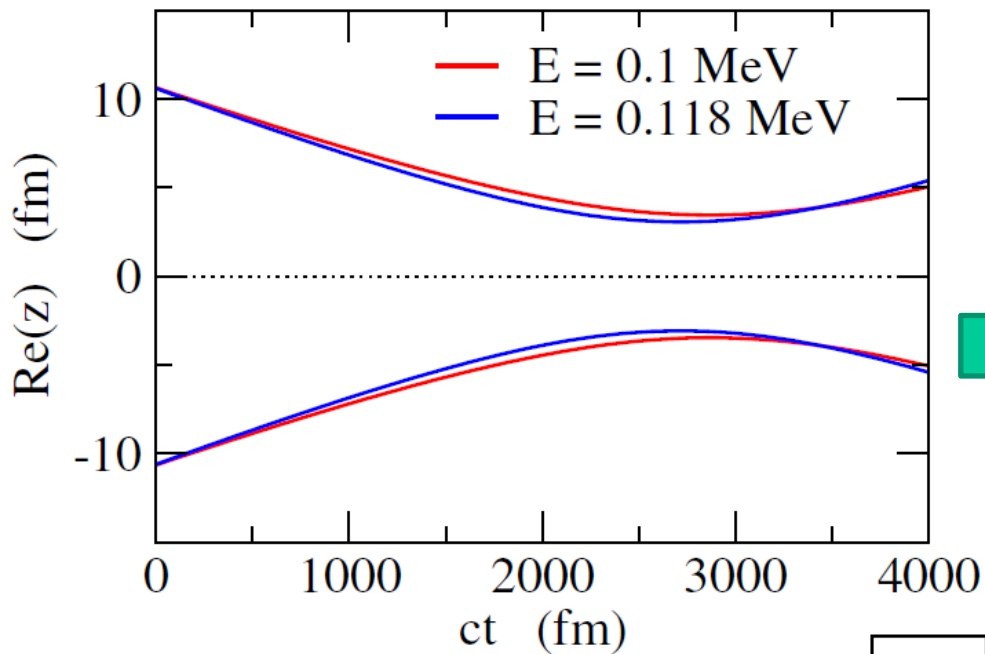
## a linear superposition of many Slater determinants



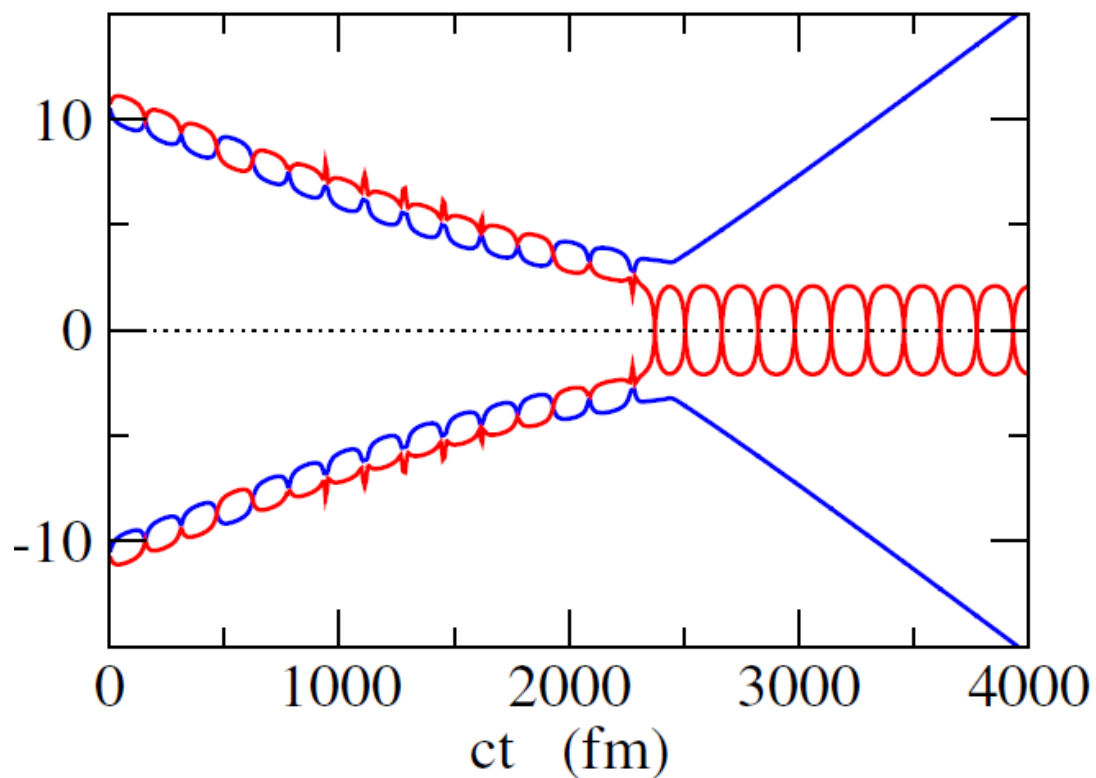
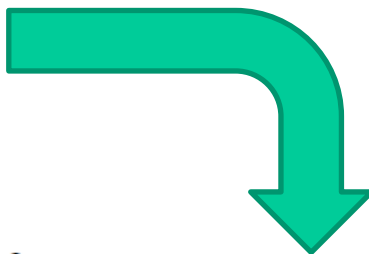
$$\Psi(t) = \sum_k \underbrace{f_k(t)} \underbrace{\Phi_{SD,k}(t)}$$

time-dependent variational principle

$$\delta \int dt \frac{\langle \Psi(t) | i\hbar \partial_t - H | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle} = 0$$



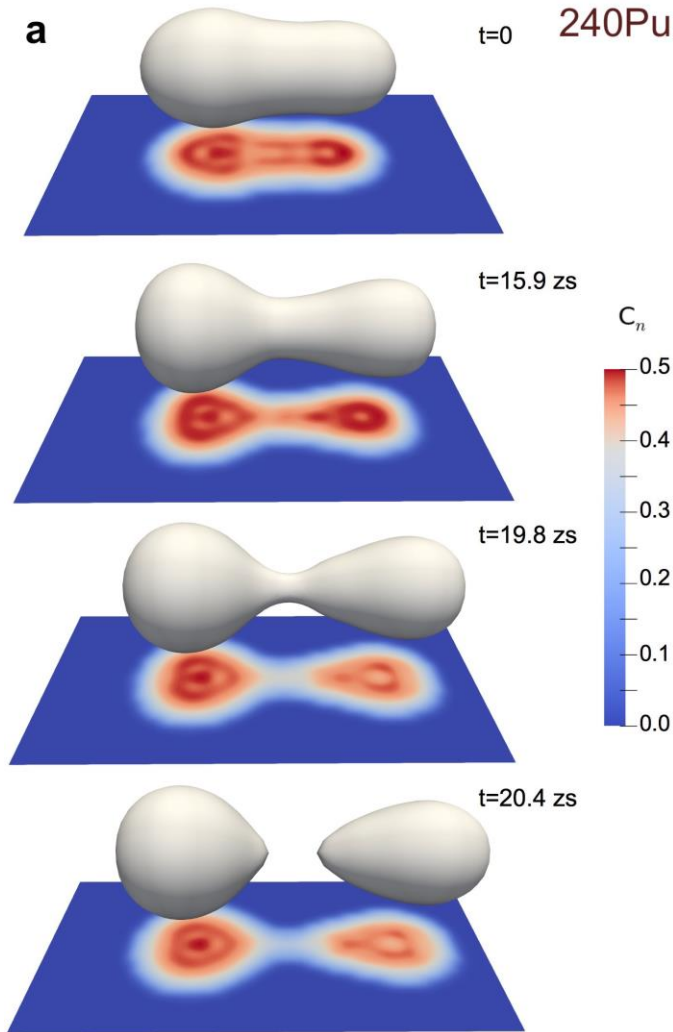
TDGCM  
 (a superposition of 2 SD)



N. Hasegawa, K.H.,  
 and Y. Tanimura,  
 PLB808, 135693 (2020)



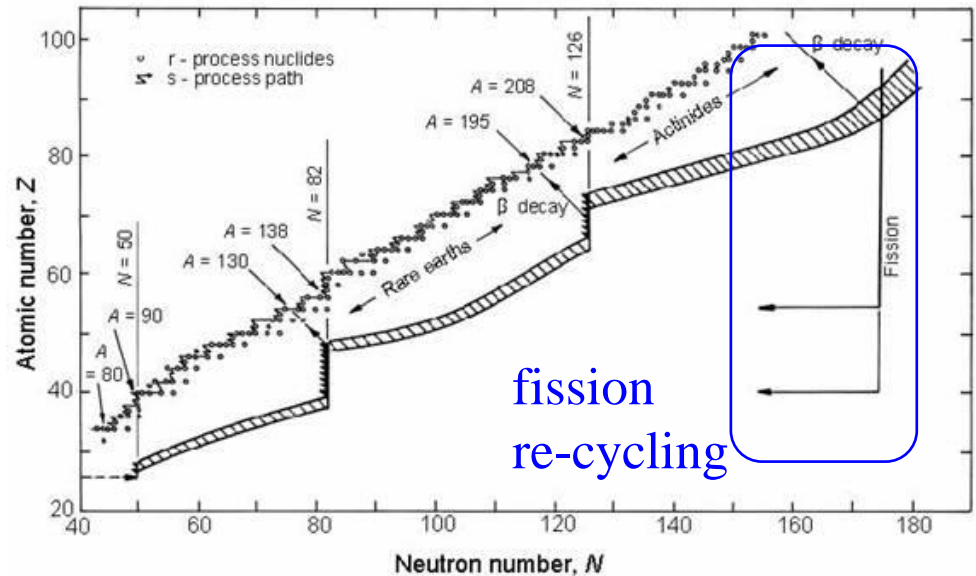
# Nuclear Fission



G. Scamps and C. Simenel,  
Nature 564 (2018) 382

➤ important role in:

- energy production
- superheavy elements
- r-process nucleosynthesis
- production of neutron-rich nuclei

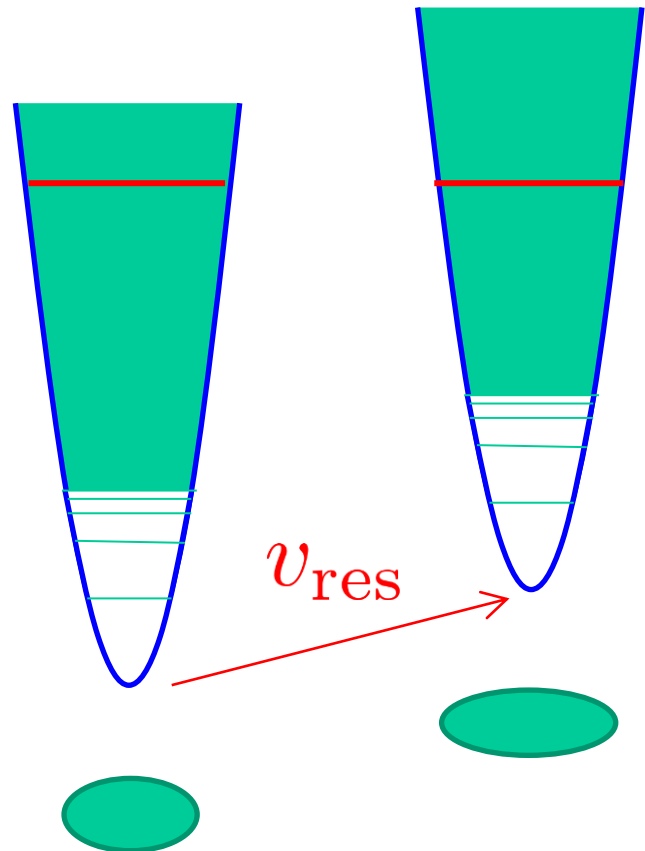
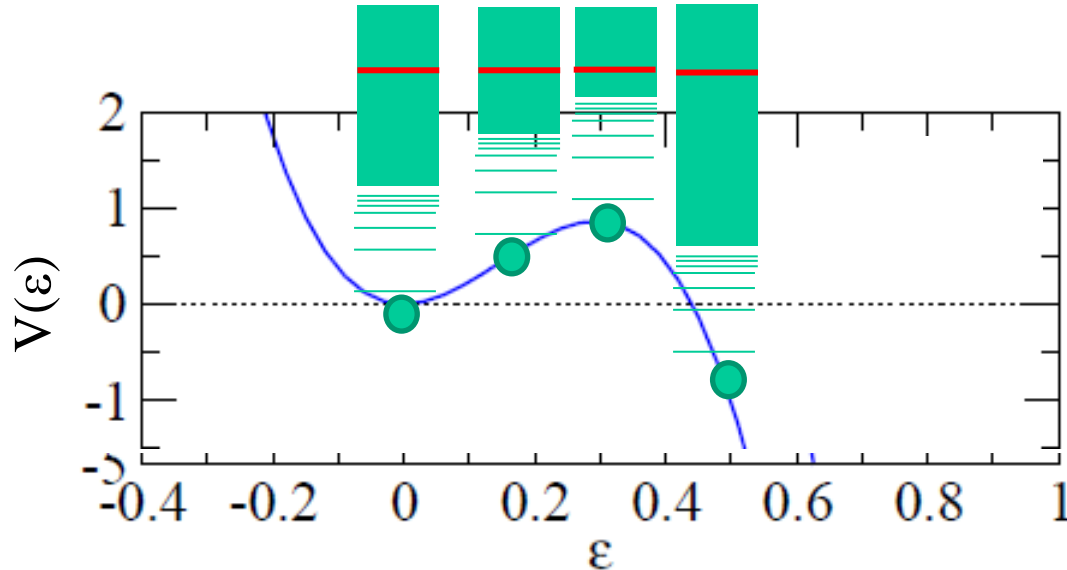


very complicated dynamics:

a microscopic understanding  
→ far from complete

# CI approach: a novel way to understand fission

K.H. and G.F. Bertsch



c.f. Generator Coordinate Method (GCM)

$$|\Psi\rangle = \int dQ f(Q) |\Phi_Q\rangle$$

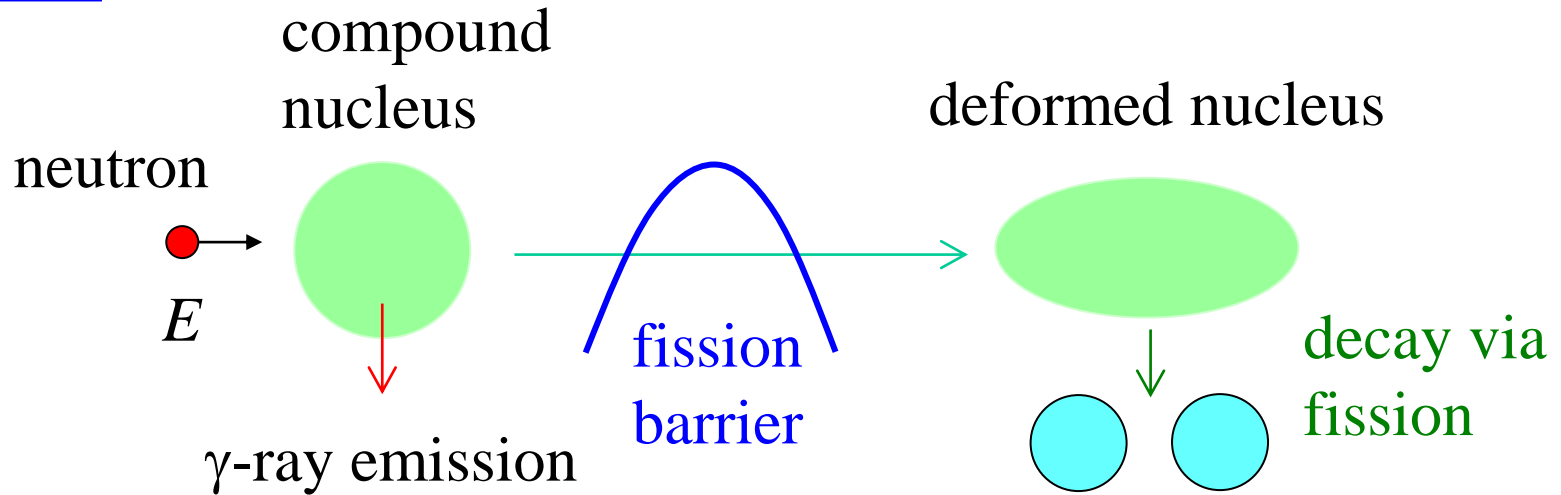
→ CI approach

$$|\Psi\rangle = \int dQ \sum_i f_i(Q) |\Phi_Q(i)\rangle$$

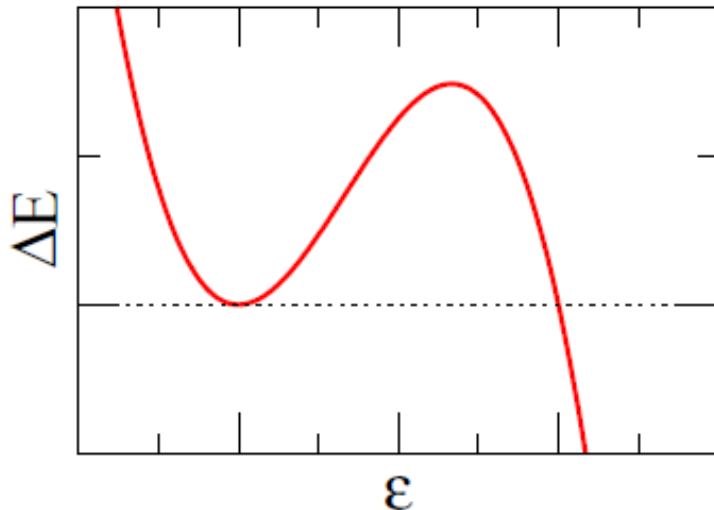
hopping due to the residual interaction

→ shape evolution

## nuclear fission



## the transition state theory

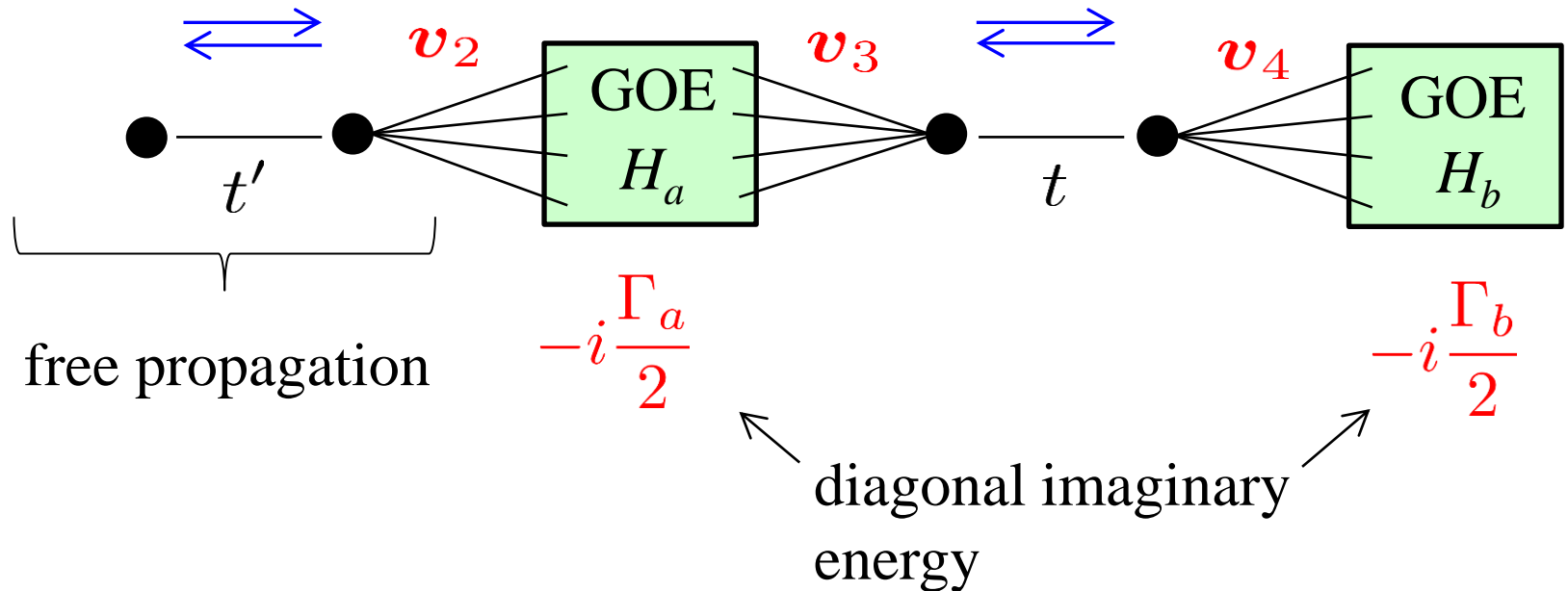
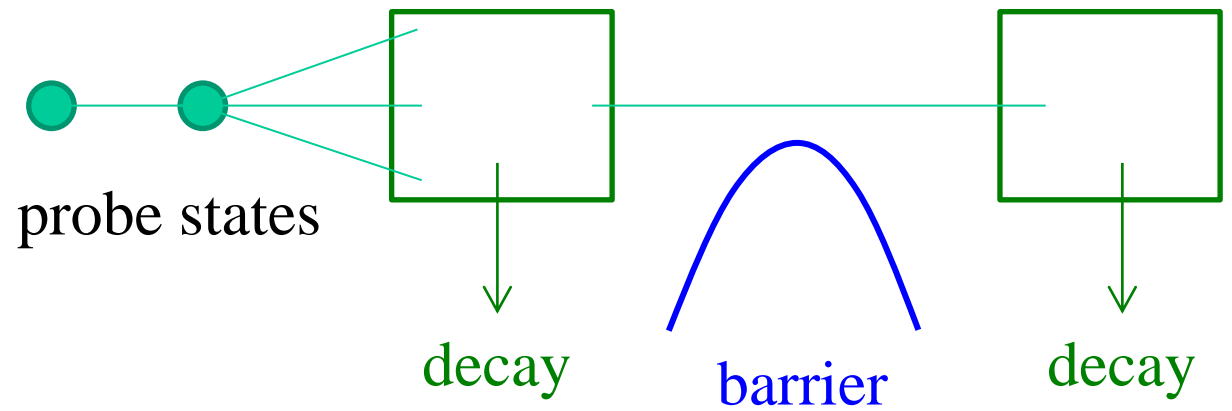


$$\Gamma_f = \frac{1}{2\pi\rho_{\text{gs}}(E^*)} \sum_c T_c$$

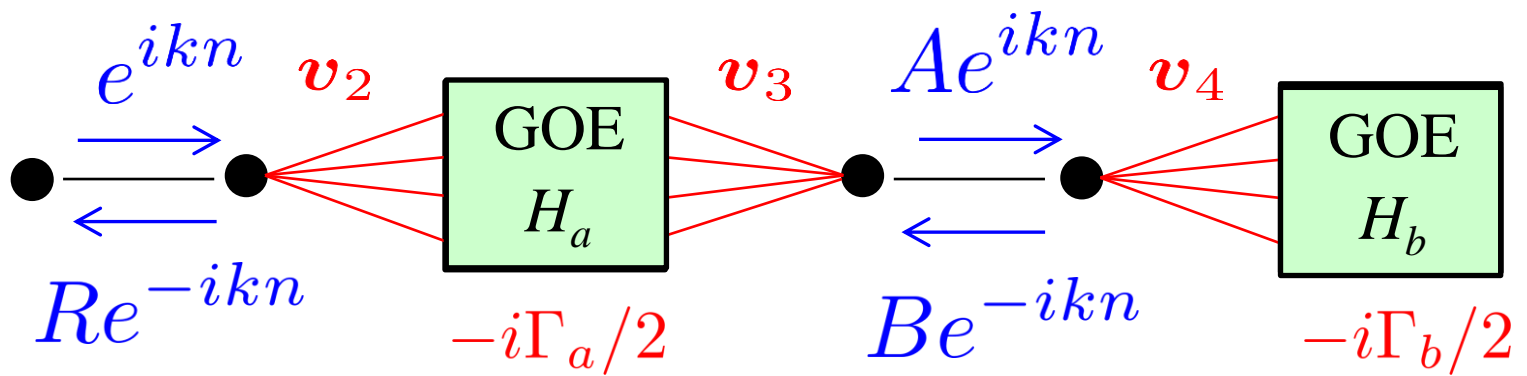
- ✓ decay dynamics: the saddle only
- ✓ the insensitvity property

Can one derive the properties of the transition state theory based on a *microscopic* many-body Hamiltonian?

## 2 GOE Model



$v_k$  ( $k = 2, 3, 4$ ): random interactions



$$T(E) = T_a(E) + T_b(E) = 1 - |R(E)|^2$$

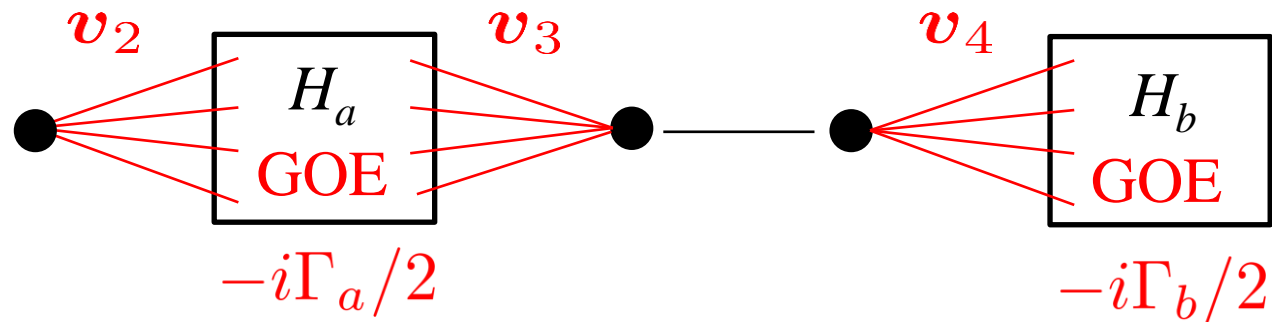
$$T_a(E) = 1 - |R(E)|^2 - |A(E)|^2 + |B(E)|^2$$

$$T_b(E) = |A(E)|^2 - |B(E)|^2$$

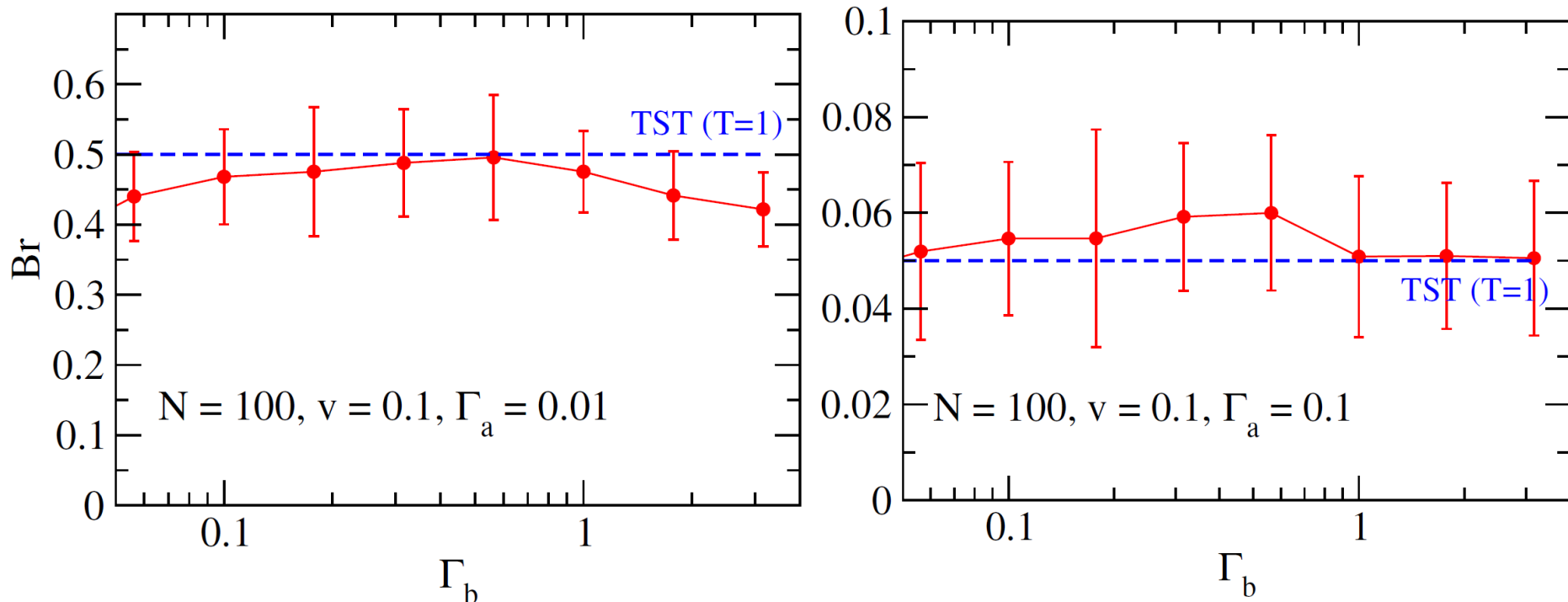
branching ratio:

$$Br = \frac{\int dE T_b(E)}{\int dE T_a(E)}$$

## branching ratios



the average and the variance with 20 ensembles



Branching ratios: insensitive to  $\Gamma_b$  ← the main assumption of the TST

the first realization of TST with a many-body Hamiltonian

# Summary

## Nuclear Reactions: a variety of many-body dynamics

Recent developments  
in nuclear fusion reactions

National Nuclear Data Center

Towards microscopic understanding

- TDGCM
- CI approach  
(nuclear fission)

Heavy and Superheavy  
nuclei

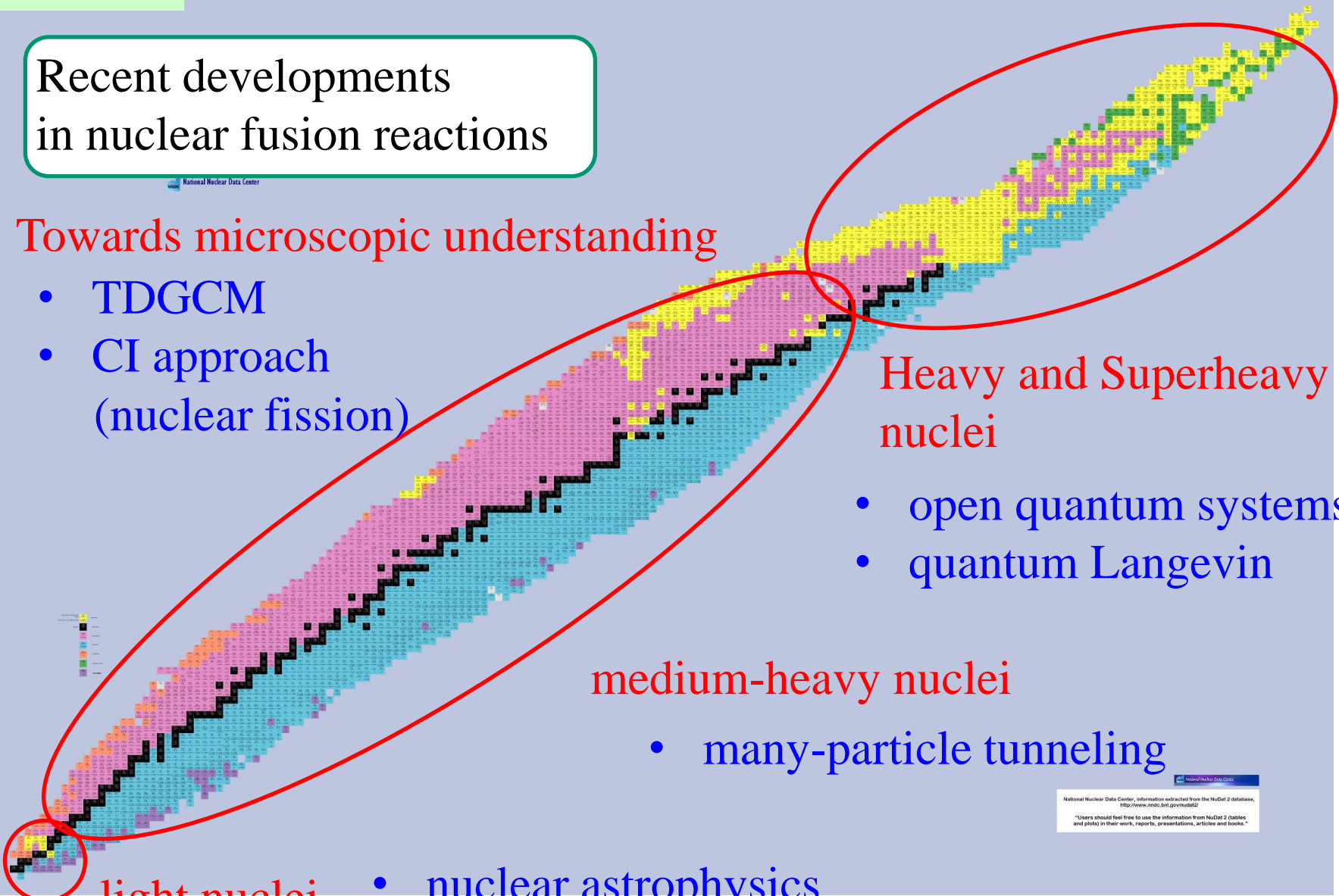
- open quantum systems
- quantum Langevin

medium-heavy nuclei

- many-particle tunneling

light nuclei

- nuclear astrophysics
- resonances



National Nuclear Data Center, information extracted from the NuDat 2 database,  
<http://www.nndc.gov/nudat2/>  
"Users should feel free to use the information from NuDat 2 (tables  
and plots) in their work, reports, presentations, articles and books."