TANTHCO Seminar, 2022.7.28

Faddeev calculations of three-neutron systems

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1. Introduction

Isospin of three-nucleon (3N) systems $\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2} = \frac{1}{2}$ or $\frac{3}{2}$

	$\boldsymbol{nnn}$	$\boldsymbol{nnp}$	$\boldsymbol{npp}$	$\boldsymbol{ppp}$
T	T_z			
$T = \frac{1}{2}$		$-\frac{1}{2}$ $[\,^3\mathrm{H}, nd]$	$+\frac{1}{2}$ $[\,^3\mathrm{He}, pd]$	
$T = \frac{3}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{3}{2}$

- Mostly studied 3N systems: Examination of “bare” nucleon-nucleon (pp and pn) force models.
- Rigorous 3N calculations assure the existence of 3N forces

→ applied to heavier nuclei

1. Introduction

- What can we learn from the study of $T = \frac{3}{2}$ 3N systems (nnn, ppp) ?

  -- direct information of nn-force and $T = \frac{3}{2}$ 3N forces

  → apply to neutron-rich nuclei, neutron matter (neutron star)

- How to study $T = \frac{3}{2}$ 3N systems (nnn, ppp):

  -- No bound state

  -- Final state of reactions: e.g., ${}^{3}\mathrm{He}(\pi^-, \pi^+)3n$, ${}^{3}\mathrm{H}(n, p)3n$

1. Introduction

- Experimental search for $3n$ resonance

$$^{3}\mathrm{He}(\pi^-,\pi^+)3n,\ ^{7}\mathrm{Li}(n,3n),\ ^{7}\mathrm{Li}(^{7}\mathrm{Li},^{11}\mathrm{C})3n,\ ^{7}\mathrm{Li}(^{11}\mathrm{B},^{15}\mathrm{O})3n,\ldots$$

Mostly negative, but a few positive results

- Experimental results that suggested the existence of $4n$ resonant state:

$(^{14}\mathrm{Be},^{10}\mathrm{Be}+4n)$[2002], $^{4}\mathrm{He}(^{8}\mathrm{He},^{8}\mathrm{Be})$[2016], $^{7}\mathrm{Li}(^{7}\mathrm{Li},^{10}\mathrm{C})$[2022], $^{8}\mathrm{He}(p,p\ ^{4}\mathrm{He})$[2022]

Refs:

Marqués et al., PRC**65** (2002), Kisamori et al., PRL **116** (2016), Faestermann et al., PLB **824** (2022), M. Duer et al., Nature **606** (2022)

- Theoretical studies on $3n$ & $4n$ systems → contradictory results

- Review:

Marqués & Carbonell (2021). Euro. Phys. J. A **57** (2021) 105. https://doi.org/10.1140/epja/s10050-021-00417-8

In this presentation:

- Quick review of theoretical calculations of $3n$ & $4n$ systems

- Theoretical method to study $3n$ continuum state [Response function, Faddeev method]

- Results of $3n$
  Ref.: S. Ishikawa, Three-neutron bound and continuum states.
  PRC **102** (2020) 034002
  https://doi.org/10.1103/PhysRevC.102.034002

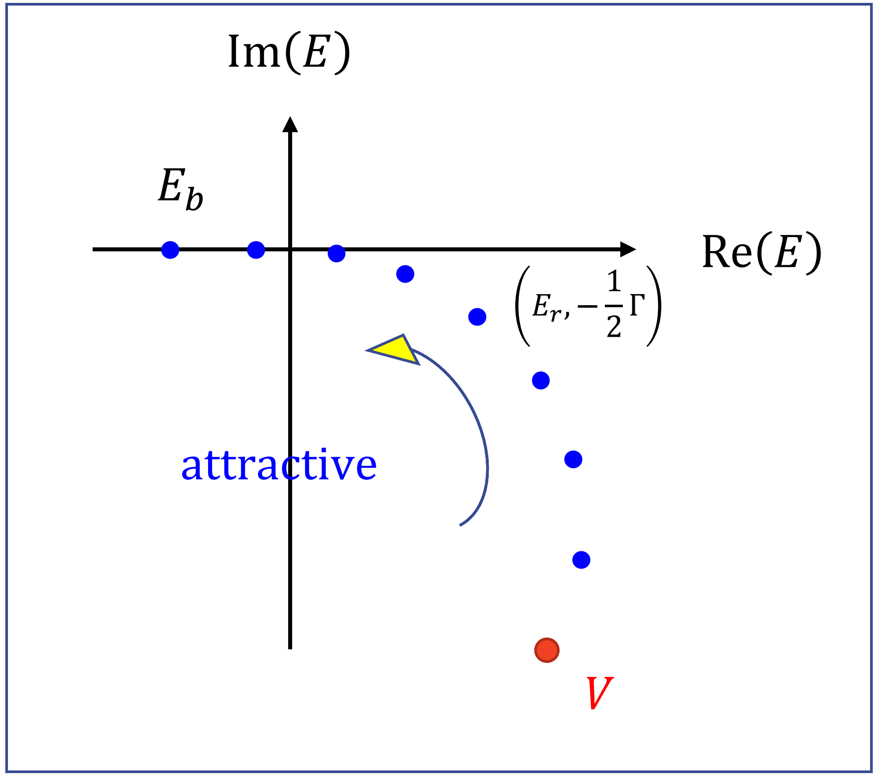
- Results of $3p$
  Ref.: S. Ishikawa, Spin-isospin excitation of ^{3}He with three-proton final state.
  Prog. Theor. and Exp. Phys. **2018** (2018) 013D03
  https://doi.org/10.1093/ptep/ptx183

2. Theoretical study for $3n$- (& $4n$-) resonance

- Realistic nucleon-nucleon potentials

  No bound state for $3n$- & $4n$-systems

- Resonance is related to a pole of t-matrix in complex energy



Pole trajectory in complex energy plane

Scattering t-matrix for complex energy ω

$$t(\omega) = V + V\frac{1}{\omega - H_0}t(\omega) = V + V\frac{1}{\omega - H_0 - V}V$$

Discrete eigen value $\quad [H_0 + V]|\Psi(z)\rangle = z|\Psi(z)\rangle$

$$t(\omega) = V + V|\Psi(z)\rangle\frac{1}{\omega - z}\langle\Psi(z)|V + \cdots$$

- Bound state: $z = E_b$, → pole at real energy $E_b < 0$
- Complex energy: $z = E_r - \frac{i}{2}\Gamma$ → pole at $\left(E_r, -\frac{1}{2}\Gamma\right)$

$3n$ studies in complex energy

- Complex energy eigenvales (1)

  - Analytic continuation with separable potentials

- Complex energy eigenvales (2)

  - Complex scaling method $x \rightarrow xe^{i\varphi}$

→ Unphysically large attractive effect is required to obtaine $3n$ bound state (or resonance)

Pole trajectory for $3n$ states with separable nn potential

PHYSICAL REVIEW C **66**, 054001 (2002)

Indications for the nonexistence of three-neutron resonances near the physical region

A. Hemmdan,[1,2,*] W. Glöckle,[1,†] and H. Kamada[3,‡]

Separable nn potential: $\langle x|V|x'\rangle = -\lambda v(x)\, v(x')$

$3n\left(\frac{3}{2}^-\right): \lambda({}^3P_2) = 3.22$

Complex E-plane

$3n\left(\frac{1}{2}^-\right): [\lambda({}^3P_2), \lambda({}^1D_2)] = [3.38, 6.88]$

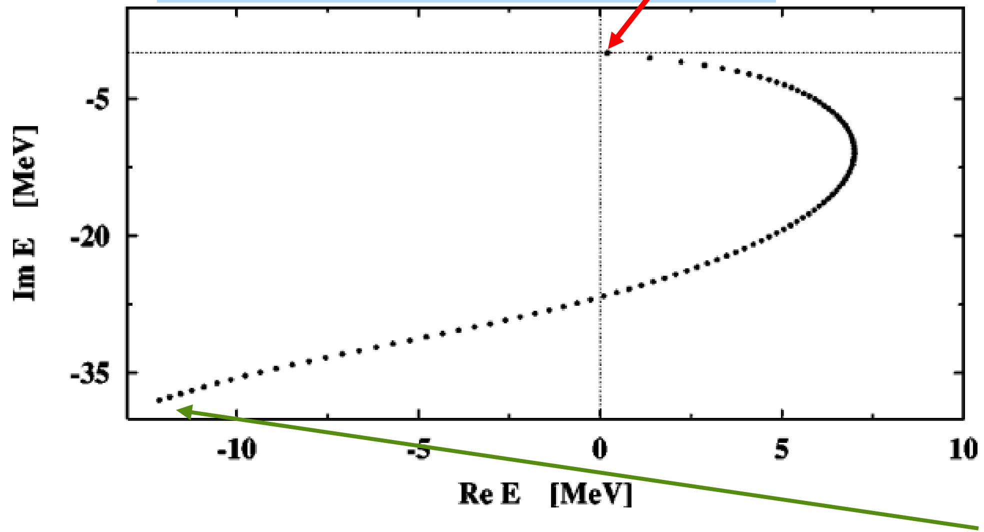


FIG. 2. The resonance pole trajectory for the state 3/2$^-$.

$\lambda = 1$

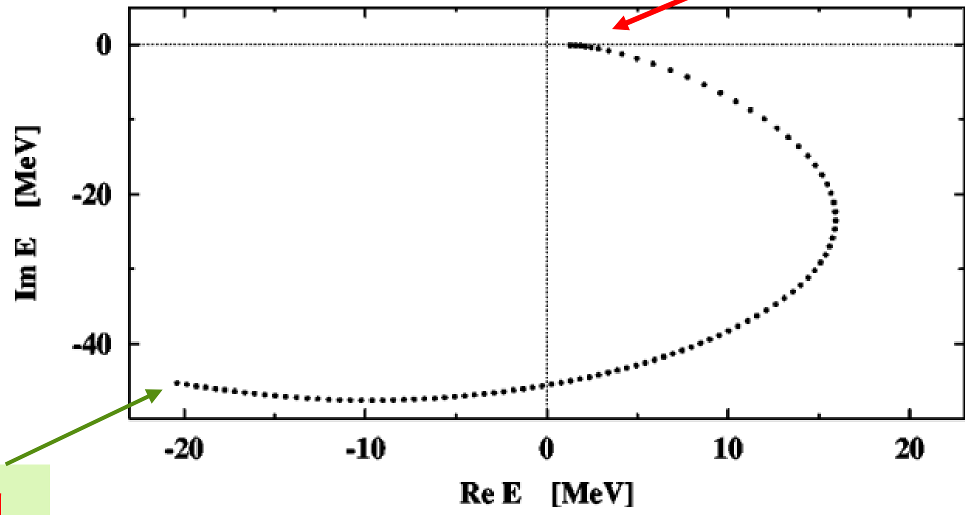


FIG. 4. The resonance pole trajectory for the state 1/2$^-$.

$3n$

Pole trajectory for $3n$ states with additional $3n$ potential

PHYSICAL REVIEW C **71**, 044004 (2005)

Three-neutron resonance trajectories for realistic interaction models

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Jaume Carbonell†

$$V_{ijk}\left(T=\frac{3}{2}\right)$$

$$V_{3n} = -W\frac{e^{-\frac{\rho}{\rho_0}}}{\rho}, \quad \text{with } \rho = \sqrt{x_{ij}^2 + y_{ij}^2} \tag{13}$$

with $\rho_0 = 2$ fm. In this way, dineutron physics is not affected.

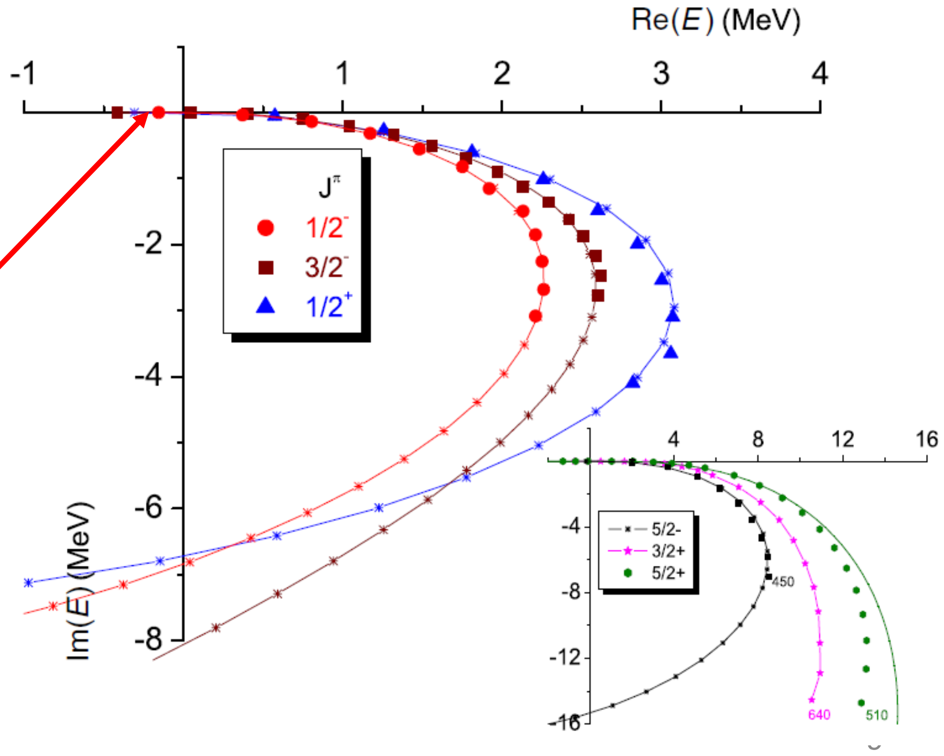
$$3n(J^\pi)$$

TABLE V. Critical strengths W_0 in MeV fm of the phenomenological Yukawa-type force of Eq. (13) required to bind the three neutron in various states. Parameter ρ_0 of this force was fixed to 2 fm. W' are the values at which three-neutron resonances become subthreshold ones, whereas B_{trit} are such 3*NF* corresponding triton binding energies in MeV.

J^π	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
W_0	307	1062	809	515	413	629
W'	152	–	329	118	146	277
B_{trit}	21.35	–	44.55	17.72	20.69	37.05

$3n\left(T=\frac{3}{2}\right)$

$nnp\left(T=\frac{1}{2}\right)$



$3n$ and $4n$ studies at real energy

- Neutrons confined in a trapping potential:

$$W(r_i) = V_0 \frac{1}{1 + e^{(r_i - R)/a_{\mathrm{WS}}}}$$

Extrapolate to real world [Strength $V_0 \to 0$]

→ Existence of $3n$ and $4n$ resonance

Energy for $4n(0^+)$ states

VOLUME 90, NUMBER 25 PHYSICAL REVIEW LETTERS week ending 27 JUNE 2003

Can Modern Nuclear Hamiltonians Tolerate a Bound Tetraneutron?

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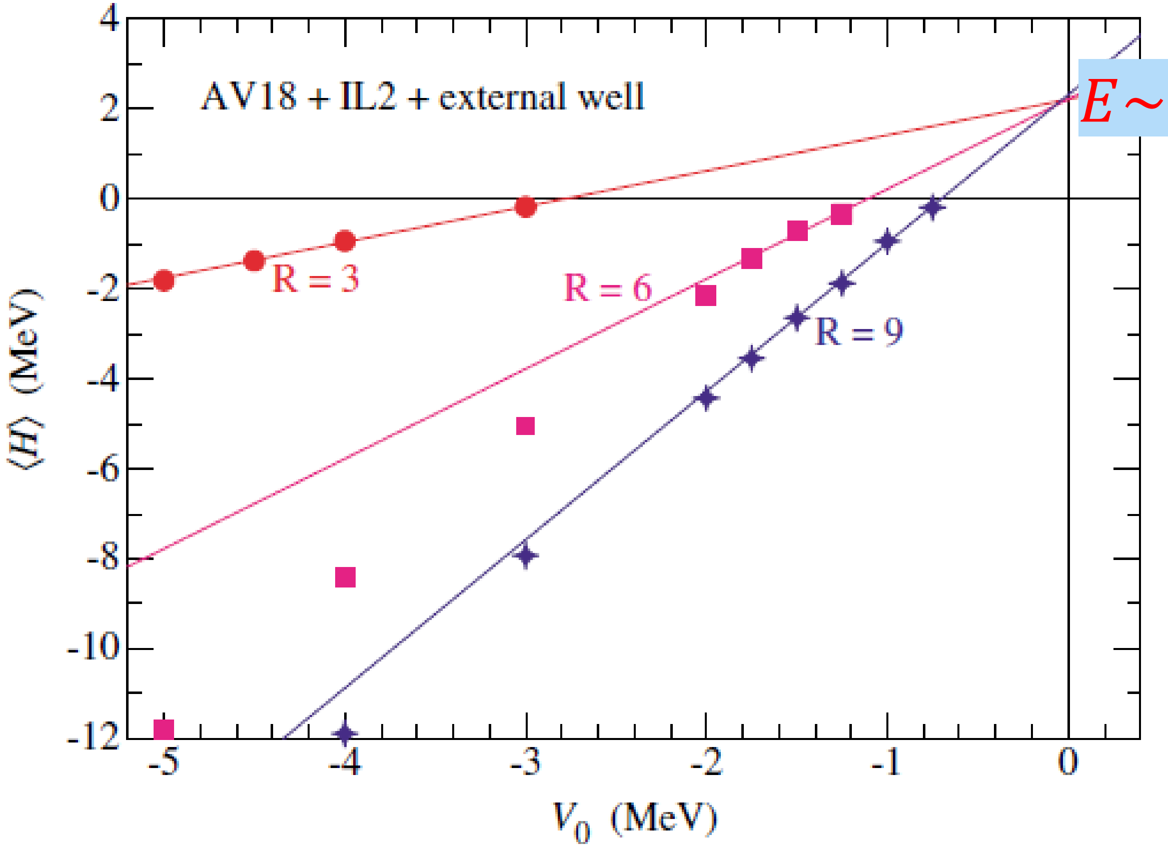
$4n$



$E\sim 2$ MeV !?

Green’s function Monte Carlo (GFMC) calculations

Artificial external wells of Woods-Saxon with (range, strength)=(R, V_0)

$$W(r_i) = V_0 \frac{1}{1 + e^{(r_i - R)/a_{\mathrm{WS}}}}$$

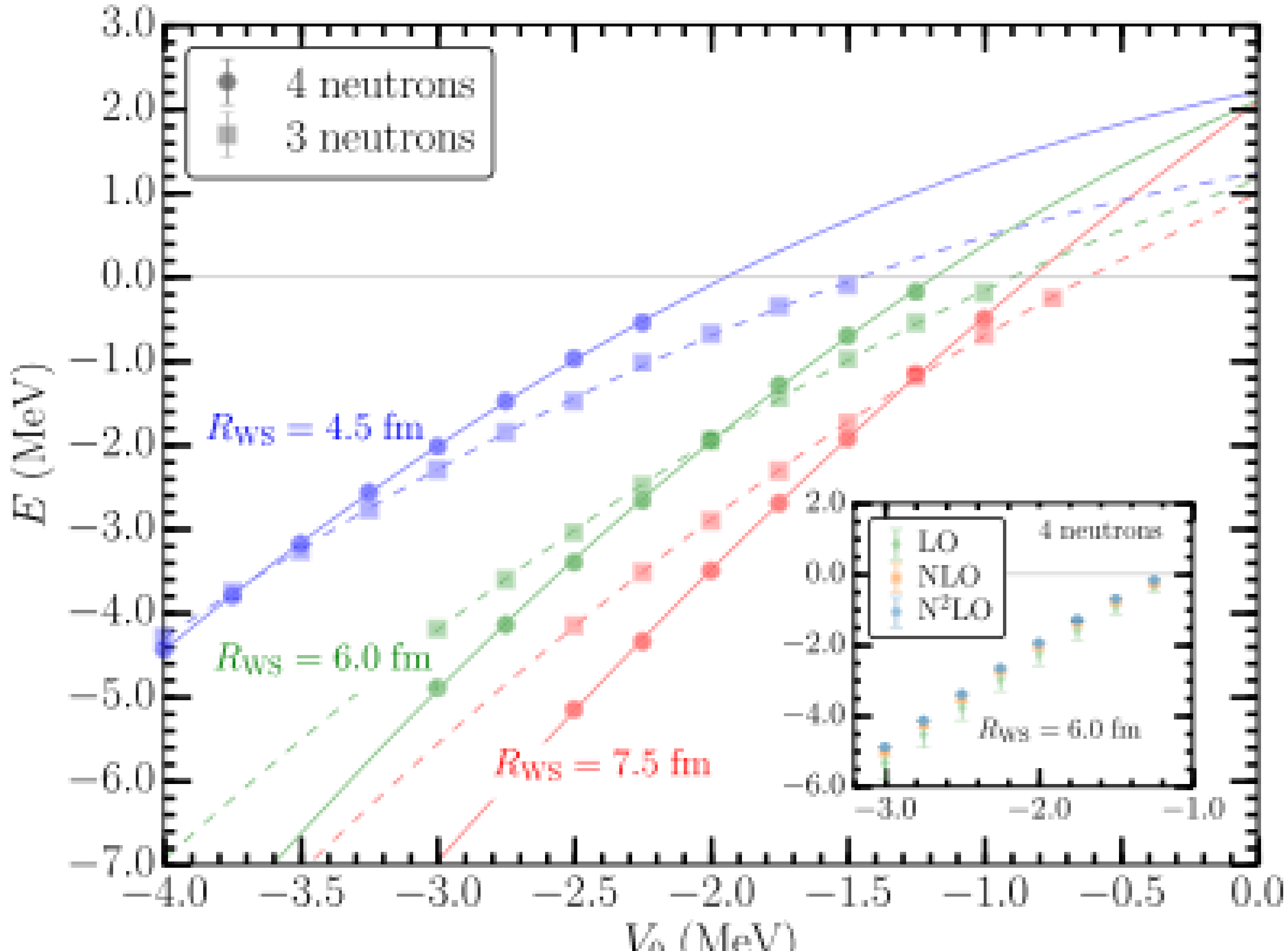
$$a_{\mathrm{WS}} = 0.65 \text{ fm}$$

Energies for $3n$ and $4n$ states

PRL **118**, 232501 (2017) PHYSICAL REVIEW LETTERS week ending 9 JUNE 2017

Is a Trineutron Resonance Lower in Energy than a Tetraneutron Resonance?

S. Gandolfi,[1,*] H.-W. Hammer,[2,3,†] P. Klos,[2,3,‡] J. E. Lynn,[2,3,§] and A. Schwenk[2,3,4,∥]



4n $E_{4n} \sim 2$ MeV

3n $E_{3n} < E_{4n}$

Monte Carlo method:

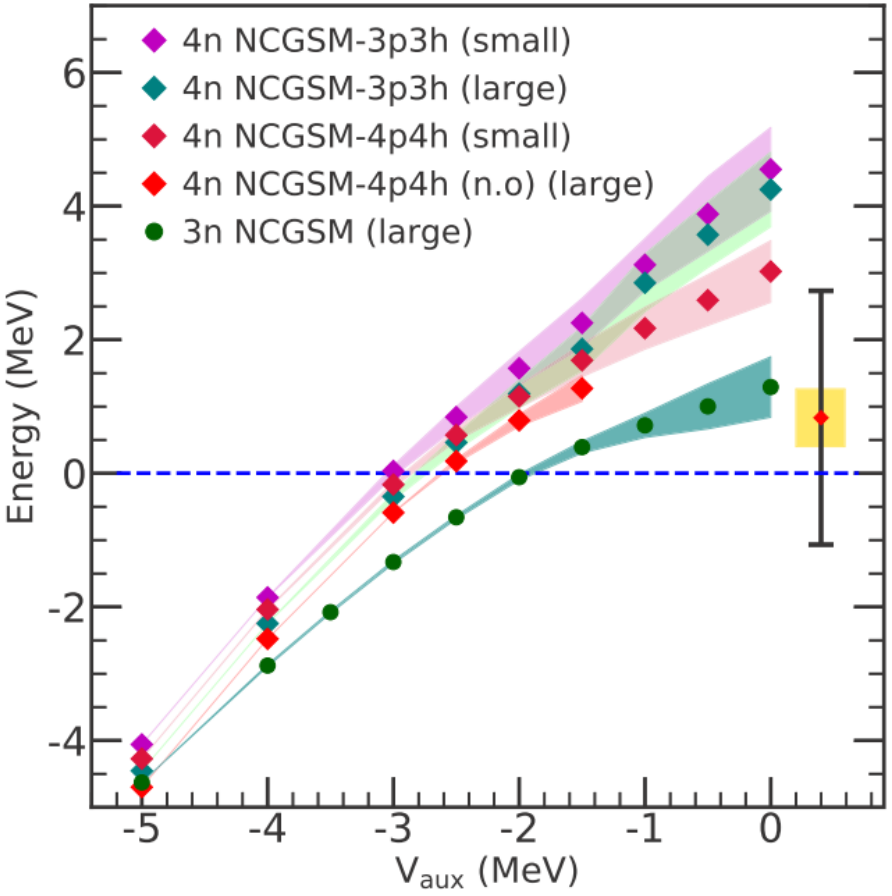
$$\langle \mathbf{R}S|\Psi_V\rangle = \langle \mathbf{R}S|\left(\prod_{i<j} f^c(r_{ij})\right)\left(1 + \sum_{i<j} F_{ij} + \sum_{i<j<k} F_{ijk}\right)|\Phi_{JM}\rangle,$$

Energies for $3n$ and $4n$ states

PHYSICAL REVIEW C **100**, 054313 (2019)

***Ab initio* no-core Gamow shell-model calculations of multineutron systems**

J. G. Li,[1] N. Michel,[2,3] B. S. Hu,[1] W. Zuo,[2,3] and F. R. Xu[1,*]



$4n$

$E_{4n} = 3\sim5$ MeV

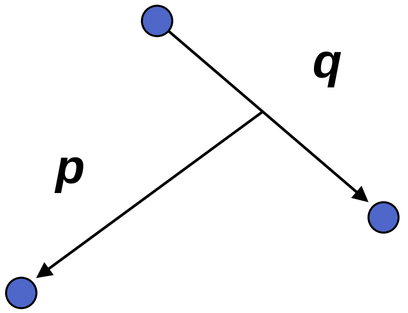
$3n$

$E_{3n} < E_{4n}$

Ab initio no-core Gamow shell model

3. How to study 3-body system without 2-body bound state

Notations:



- Total Hamiltonian (only 2NF for simplicity)

$$H = H_0 + V_1 + V_2 + V_3$$

$$V_1 = V_{23} \qquad etc. \text{ (odd man out notation)}$$

- (Asymptotic) 3-body states are specified by momentum-variables $\vec{q}, \vec{p}$ $\quad |\vec{q}, \vec{p}\rangle$

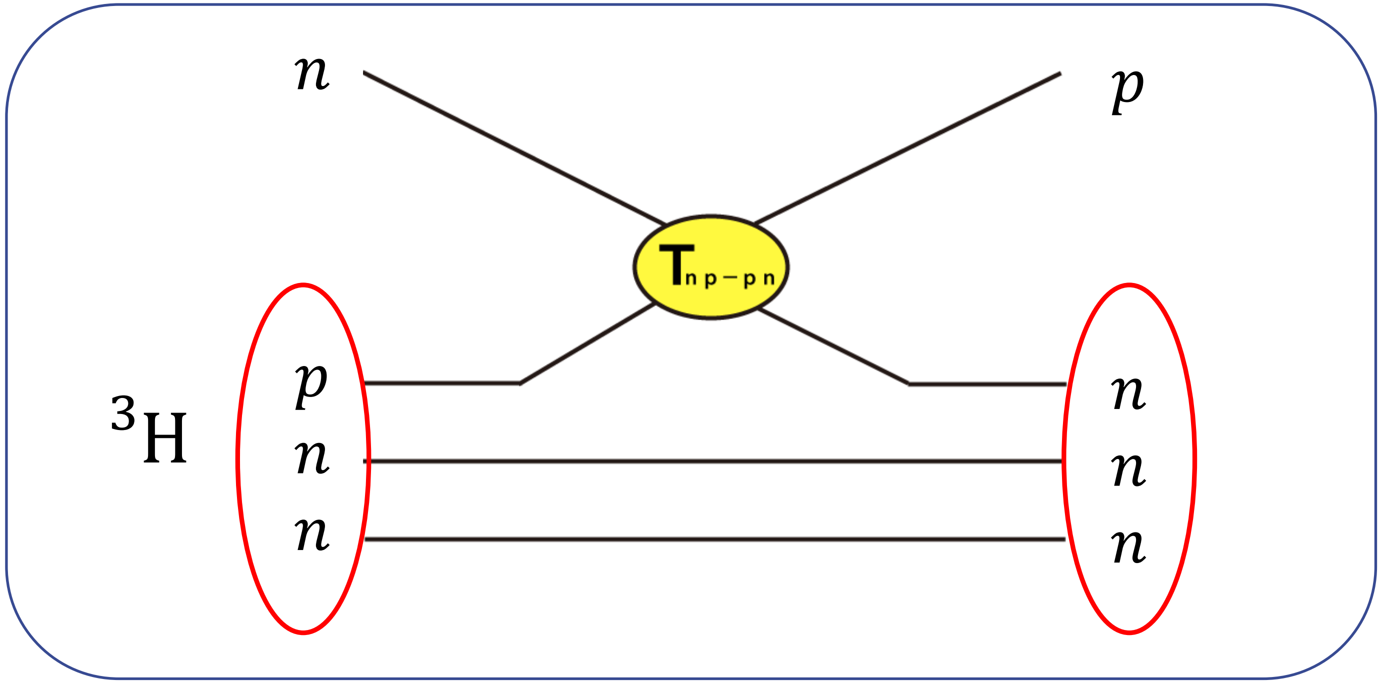
$$H_0 |E; \vec{q}, \vec{p}\rangle = E |E; \vec{q}, \vec{p}\rangle, \qquad E = \frac{\hbar^2}{m} q^2 + \frac{3\hbar^2}{4m} p^2 = E_q + E_p$$

- Eigenstate of 3-body Hamiltonian with going (+) / incoming (−) boundary conditions:

$$H \left| \Psi_{\vec{q}\vec{p}}^{(\pm)}(E) \right\rangle = E \left| \Psi_{\vec{q}\vec{p}}^{(\pm)}(E) \right\rangle$$

Reactions to study $3n$ & $3p$ states

- Reactions to produce $3n$ (or $3p$) state with simple reaction mechanism
  e.g., $^{3}\mathrm{H}(n,p)3n$ $^{3}\mathrm{He}(p,n)3p$
- In PWIA
  Two processes: $n+p \to p+n$, $^{3}\mathrm{H} \to nnn$ (or $^{3}\mathrm{He} \to ppp$)
  Transition amplitude: $T \propto t_{np\to pn} \times \left\langle \Psi^{(-)}_{\vec{q}\vec{p}}(E) \middle| \hat{O} \middle| \Psi_b \right\rangle$



Response functions

- In PWIA, the cross section can be written in terms of response function:

$$R_{\hat{O}}(E) = \int d\vec{q}\, d\vec{p} \left| \left\langle \Psi^{(-)}_{\vec{q}\vec{p}}(E_q + E_p) \middle| \hat{O} \middle| \Psi_b \right\rangle \right|^2 \delta(E - E_q - E_p)$$

$$= -\frac{1}{\pi} \mathrm{Im} \left\langle \Psi_b \middle| \hat{O}^\dagger \frac{1}{E + i\varepsilon - H} \hat{O} \middle| \Psi_b \right\rangle$$

- If the system has a complex energy eigen value, $E_r - \frac{i}{2}\Gamma$:

$$H|\Psi\rangle = \left(E_r - \frac{i}{2}\Gamma\right)|\Psi\rangle \quad \rightarrow \quad R_{\hat{O}}(E) = \frac{R_r}{\pi} \frac{\frac{\Gamma}{2}}{(E-E_r)^2 + \left(\frac{1}{2}\Gamma\right)^2}$$

- When the complex energy is close to real axis (i.e. Γ is small enough) so that $R_{\hat{O}}(E)$ has a peak around $E = E_r$, it is called as a resonance peak.

Note: $\hat{O}_c = \sum_{i=1}^{3} e^{i\vec{Q}\cdot\vec{r}_i} t_i^{(-)}$, $\hat{O}_L = \sum_{i=1}^{3} e^{i\vec{Q}\cdot\vec{r}_i} (\hat{Q}\cdot\hat{\sigma}_i) t_i^{(-)}$, $\hat{O}_T = \sum_{i=1}^{3} e^{i\vec{Q}\cdot\vec{r}_i} (\hat{Q}\times\hat{\sigma}_i) t_i^{(-)}$

Calculation of the Response functions

$$R_{\hat{O}}(E) = \int d\vec{q}d\vec{p} \left| \left\langle \Psi_{\vec{q}\vec{p}}^{(-)}(E_q + E_p) \middle| \hat{O} \middle| \Psi_b \right\rangle \right|^2 \delta(E - E_q - E_p)$$

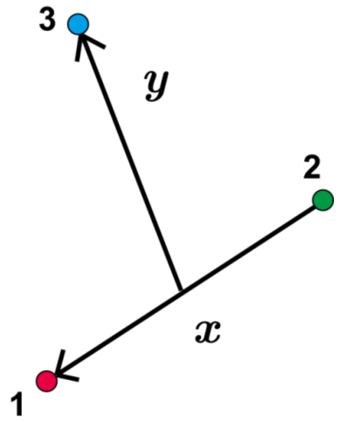
- Use the Green's function method to avoid to calculate $\Psi_{\vec{q}\vec{p}}^{(-)}(E = E_q + E_p)$ for all possible combinations of E_q and E_p for a given E:

$$R_{\hat{O}}(E) = -\frac{1}{\pi} \mathrm{Im} \left\langle \Psi_b \middle| \hat{O}^\dagger \frac{1}{E + i\varepsilon - H} \hat{O} \middle| \Psi_b \right\rangle$$

- Def. $|\Psi(E)\rangle$: wave function corresponding to the process $^3\mathrm{H} \to 3n$:

$$|\Psi(E)\rangle = \frac{1}{E + i\varepsilon - H} \hat{O} |\Psi_b\rangle$$

Calculation of the Response functions



- Asymptotic form of $|\Psi(E)\rangle$

$$\langle \vec{x}\vec{y}|\Psi\rangle = \langle \vec{x}\vec{y}|\frac{1}{E+i\varepsilon-H}\hat{O}|\Psi_b\rangle \to N\frac{e^{iKR}}{R^{5/2}}\left\langle \Psi^{(-)}_{\vec{q}\vec{p}}\middle|\hat{O}\middle|\Psi_b\right\rangle$$

$$R = \sqrt{x^2 + \frac{4}{3}y^2} \qquad K = \sqrt{\frac{m}{\hbar^2}E}$$

- Once the function $|\Psi(E)\rangle$ is obtained, all of the 3-body breakup amplitudes $\left\langle \Psi^{(-)}_{\vec{q}\vec{p}}\middle|\hat{O}\middle|\Psi_b\right\rangle$ are calculated from $|\Psi(E)\rangle$.

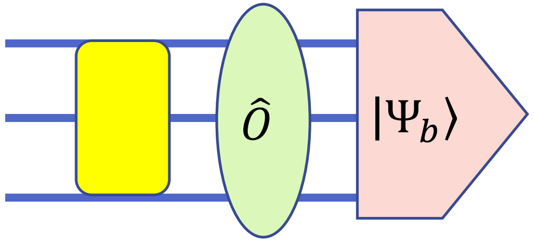
References: Faddeev calculations for $3\alpha(0^+)$ systems:
S. I. : PRC **87** (2013) 055804, PRC **90** (2014) 061604, PRC **94** (2016) 061603

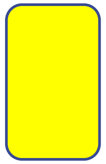
How to calculate the wave function $|\Psi(E)\rangle$

- Three-body problem under the 3-body Hamiltonian H

- Expression by the diagram

$$|\Psi(E)\rangle = \frac{1}{E+i\varepsilon-H}\hat{O}|\Psi_b\rangle =$$



-  → full 3-body dynamics including 3-body T-matrix $T(E)$

- Faddeev (1961) :
  Decompose the T-matrix with respect to interaction pair in the final state

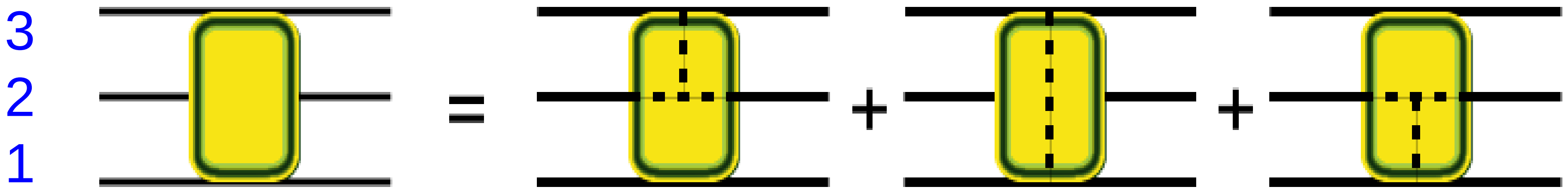
$$T(E) = T^{(1)}(E) + T^{(2)}(E) + T^{(3)}(E)$$

Apply the Faddeev theory to calculate $|\Psi(E)\rangle$

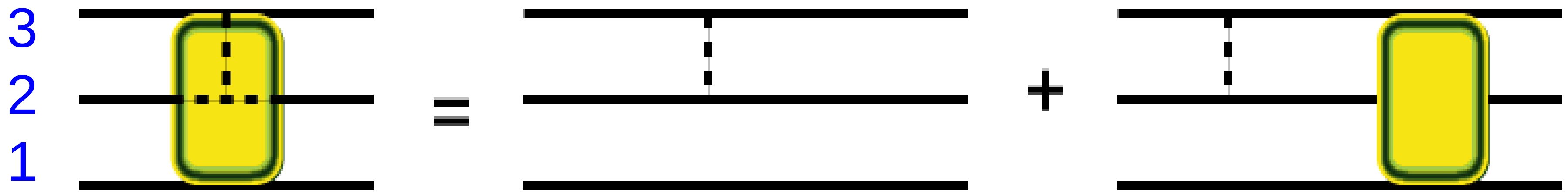
- Ref. L.D. Faddeev, "Scattering Theory for a Three-Particle System" Soviet Phys. JETP **12** (1961) 1014:

  Decompose the T-matrix with respect to interaction pair in the final state

$$T(E) = T^{(1)}(E) + T^{(2)}(E) + T^{(3)}(E)$$



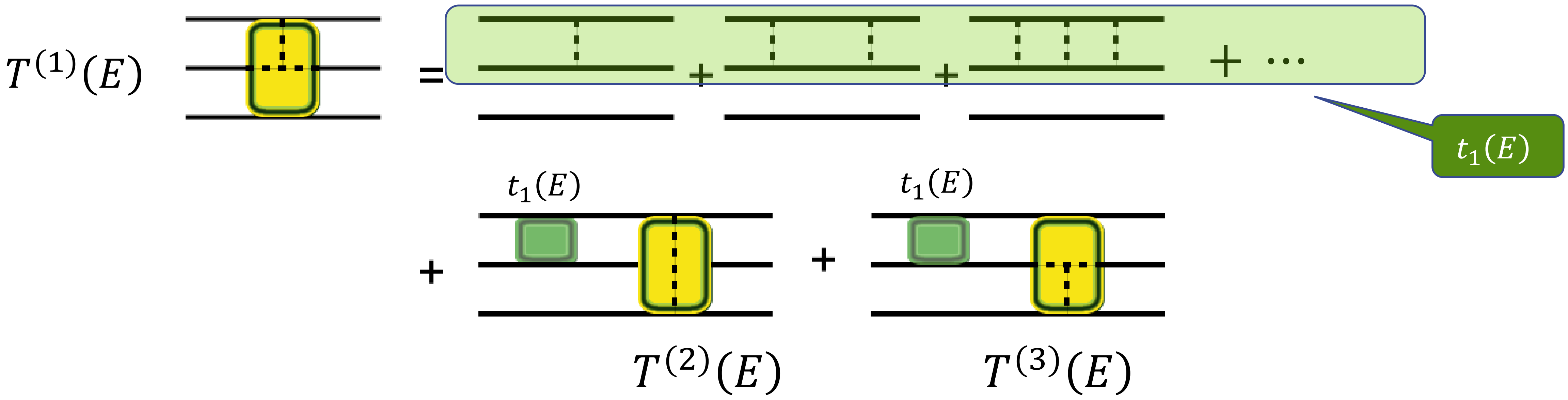
$$T^{(1)}(E) = V_1 + V_1 G_0(E) T(E)$$



Faddeev equations for T-matrix

- Multiple scattering with rearrangements for the Faddeev components $T^{(i)}(E)\ \ (i = 1,2,3)$

$$T^{(1)}(E) = t_1(E) + t_1(E)G_0(E)\left[T^{(2)}(E) + T^{(3)}(E)\right]$$



Faddeev equations for $|\Psi(E)\rangle$

- Channel Hamiltonian

$$H_i = H_0 + V_i, \qquad H = H_i + V_j + V_k$$

- In general

$$\hat{O} = \hat{O}_1 + \hat{O}_2 + \hat{O}_3$$

- Faddeev decomposition

$$|\Psi\rangle = \frac{1}{E + i\varepsilon - H}\hat{O}|\Psi_b\rangle = |\Phi_1\rangle + |\Phi_2\rangle + |\Phi_3\rangle$$

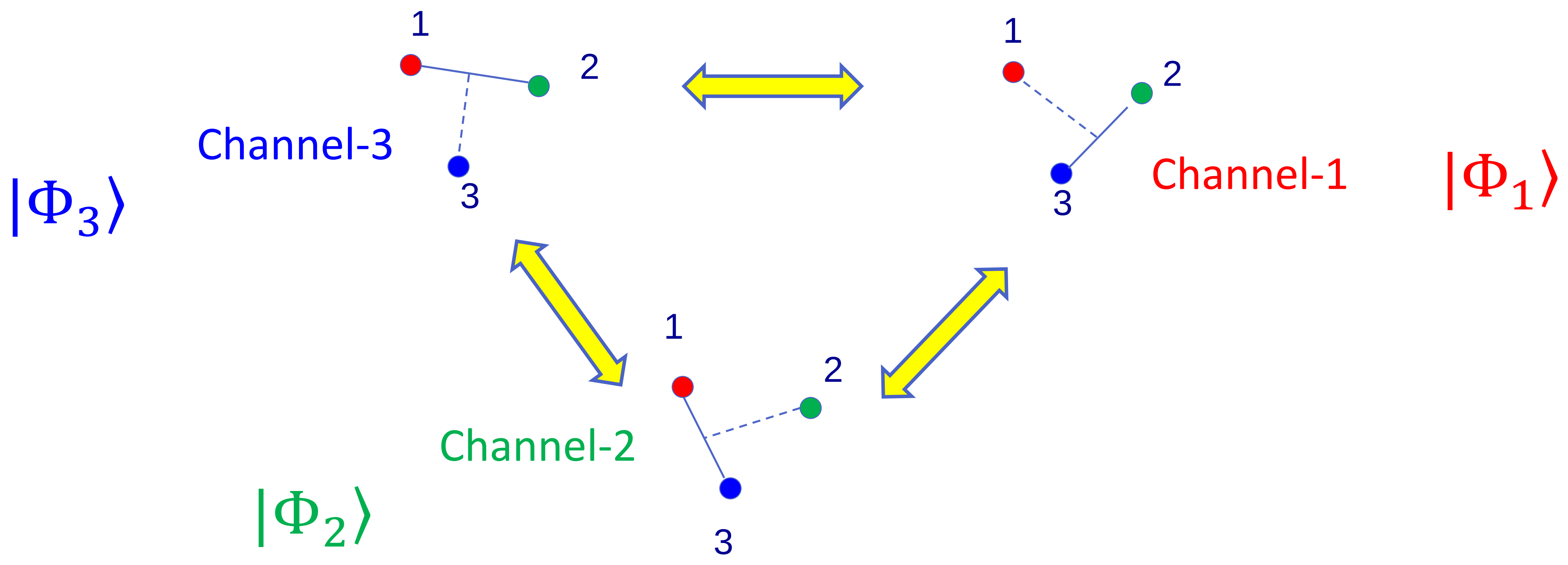
- Faddeev equations for the Faddeev components

$$|\Phi_1\rangle = \frac{1}{E + i\varepsilon - H_1}\hat{O}_1|\Psi_b\rangle + \frac{1}{E + i\varepsilon - H_1}V_1(|\Phi_2\rangle + |\Phi_3\rangle)$$

$$(1,2,3) \to (2,3,1) \to (3,1,2)$$

Multiple scattering with rearrangement

$$|\Phi_1\rangle = \frac{1}{E + i\varepsilon - H_1}\hat{O}_1|\Psi_b\rangle + \frac{1}{E + i\varepsilon - H_1}V_1(|\Phi_2\rangle + |\Phi_3\rangle)$$



4. Calculations of the response functions

Response function $R_{\hat{O}}(E,Q)$ for the transition from the $^3\mathrm{H}$ ground state to $3n\left(\frac{3}{2}^-\right)$ continuum state with $\hat{O} = \sum_{i=1}^{3} e^{i\vec{Q}\cdot\vec{r}_i}\, t_i^{(-)}$.

[0] Calculations with Argonne V18$-nn$ potential

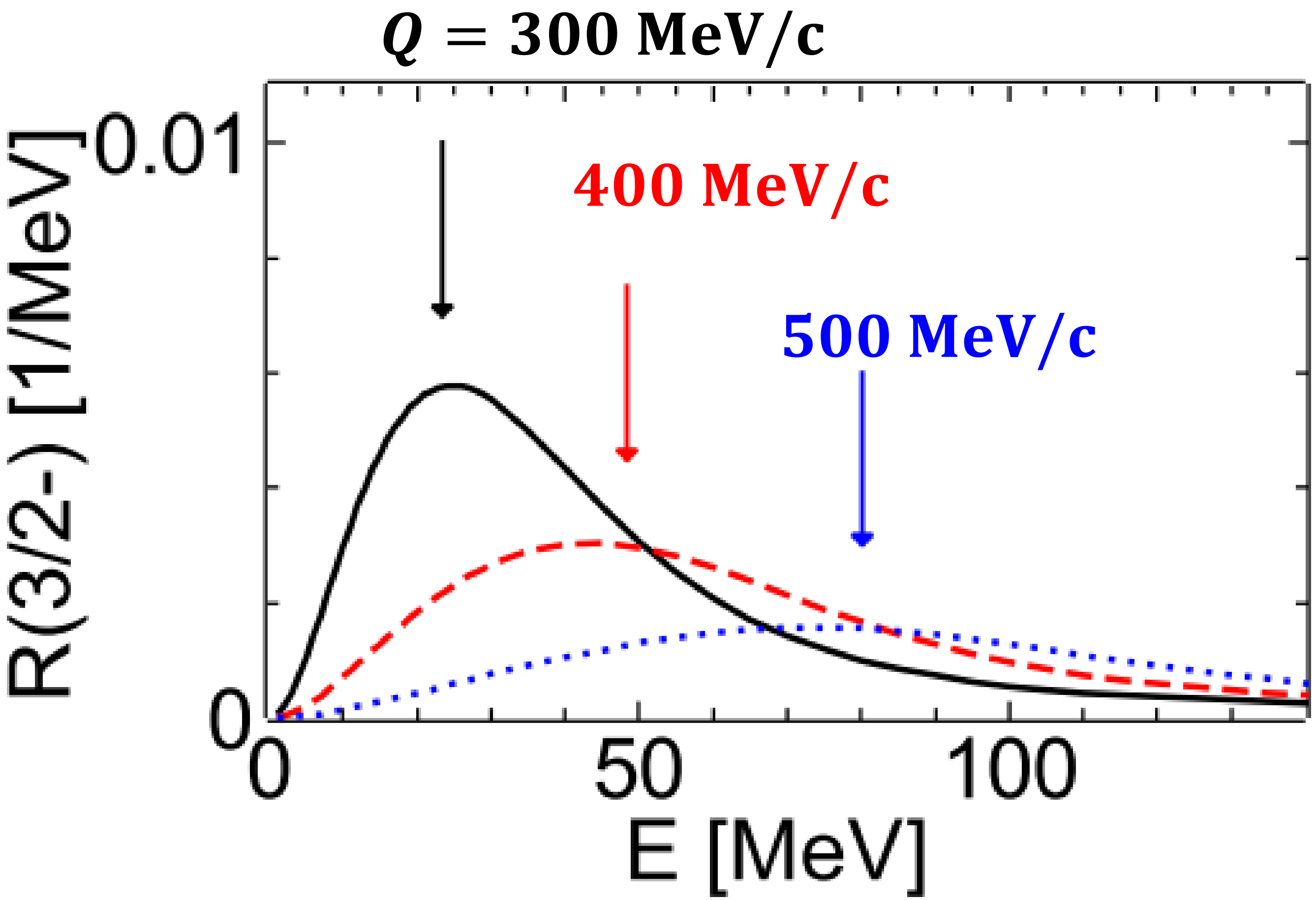
Extrapolation procedures with giving additional attractions to the $3n$ Hamiltonian

[1] Multiplying a factor to the nn potential

[2] Introducing a 3BP

[3] Additional trapping potential

[0] Calculations with AV18−nn potential



Arrows:

$$E = \frac{Q^2}{2m} - B(\,^3\mathrm{H}) - \frac{Q^2}{6m}$$

Quasifree process that the momentum Q is absorbed by one neutron.

[1] Multiplying a factor to the nn potential

- Modify the nn potential by multiplying a factor $(1-\alpha)$

$$V\left({}^{2S+1}L_J\right) \rightarrow (1-\alpha) \times V\left({}^{2S+1}L_J\right)$$

- Note: $nn\left({}^{1}S_0\right)$-state has a bound state for $\alpha < -0.08$

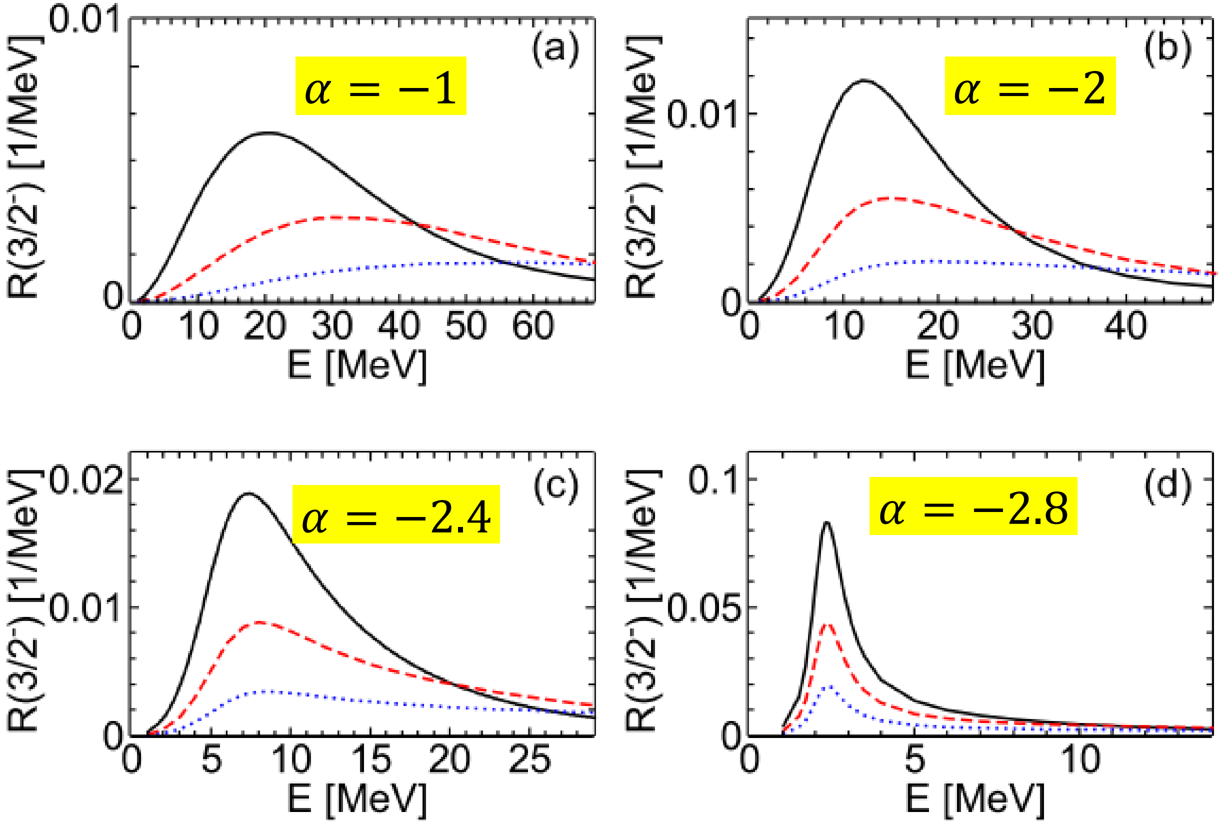
- The factor will be multiplied only to $V\left({}^{3}P_2 - {}^{3}F_2\right)$ [attractive]

$nn\left({}^{3}P_2 - {}^{3}F_2\right)$ bound state exists for $\alpha < -3.39$

$3n\left(\frac{3}{2}^{-}\right)$ bound state exists for $\alpha < -2.98$

[1] Multiplying a factor to the nn potential

$\boldsymbol{Q = 300, 400, \text{and } 500}$ **MeV/c**

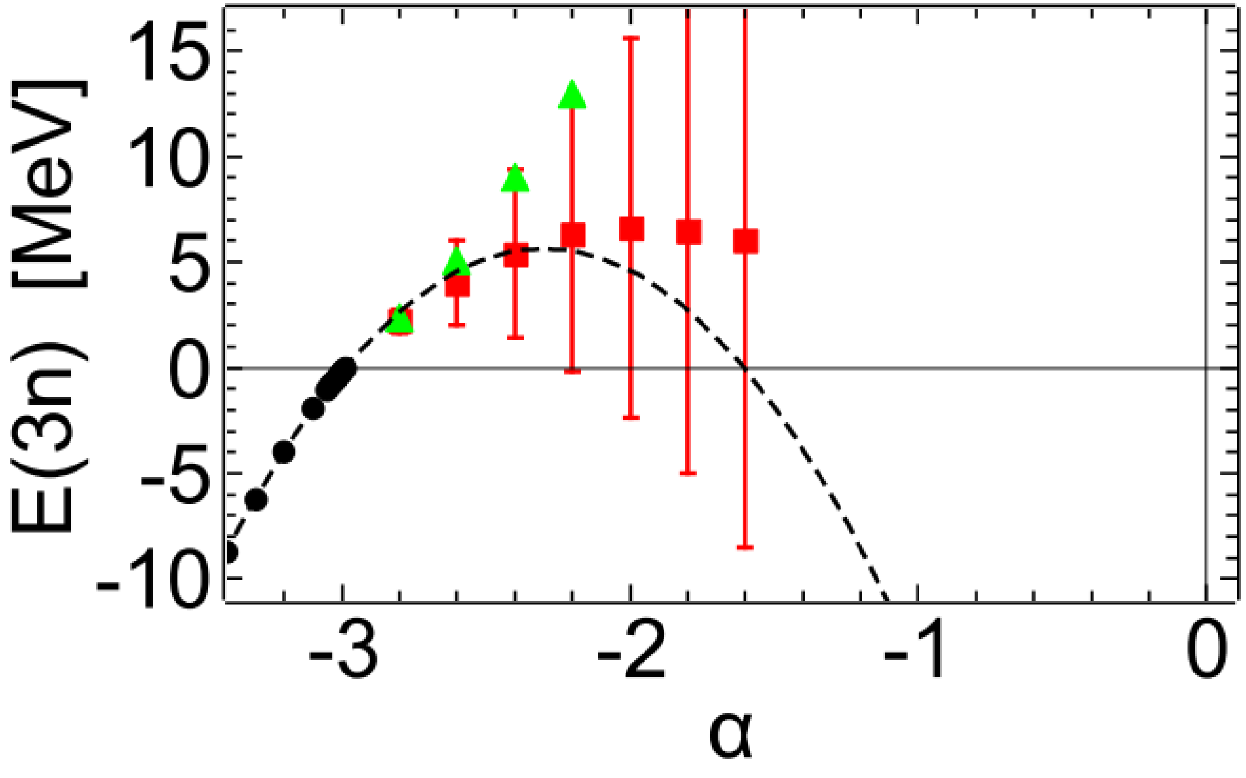


- Fitting of the response function

$$R(E) = \frac{b(E-E_r)+c\Gamma}{(E-E_r)^2+\Gamma^2/4} + a_0 + a_1(E-E_r) + a_2(E-E_r)^2$$

- Extracted values of E_r and Γ are Q-independent for $-2.7 \leq \alpha \leq -1.6$

[1] Multiplying a factor to the nn potential



● $3n$ binding energy

- - - Fitted to $3n$ binding energy

■ Extracted E_r $\left(\pm\frac{\Gamma}{2}\right)$

▲ Peak energy

$\alpha \to 0$

No pole close to the real axis

[2] Introducing a 3BP

- Three-body potential

$$W(T) = \sum_{n=1}^{2} W_n e^{-(r_{12}^2 + r_{23}^2 + r_{31}^2)/b_n^2} \hat{P}(T)$$

- Range parameters: $b_1 = 4.0\ \mathrm{fm}, b_2 = 0.75\ \mathrm{fm}$
  Short range repulsive term $W_2 = +35.0\ \mathrm{MeV}$
  [Hiyama et al., PRC93 (2016) 044004]

  Required value of W_1 for $4n(0^+)$ state to bind: $W_1 = -36.14\ \mathrm{MeV}$

- $n\left(\frac{3}{2}^-\right)$ bound state exists for $W_1 < -80\ \mathrm{MeV}$
  $\Leftrightarrow W_1 = -2.55\ \mathrm{MeV}$ to reproduce $^3\mathrm{H}$ binding energy

Pole trajectory for $3n$ states and energy for $4n(0^+)$ states

PHYSICAL REVIEW C **93**, 044004 (2016)

Possibility of generating a 4-neutron resonance with a $T = 3/2$ isospin 3-neutron force

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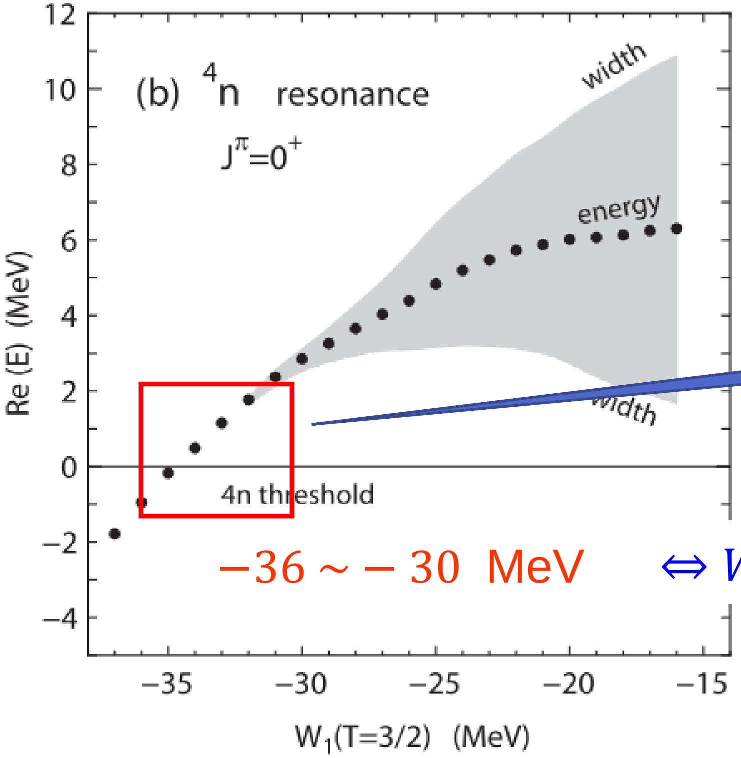
M. Kamimura

$V_{ijk}\left(T = \frac{3}{2}\right)$

$4n$



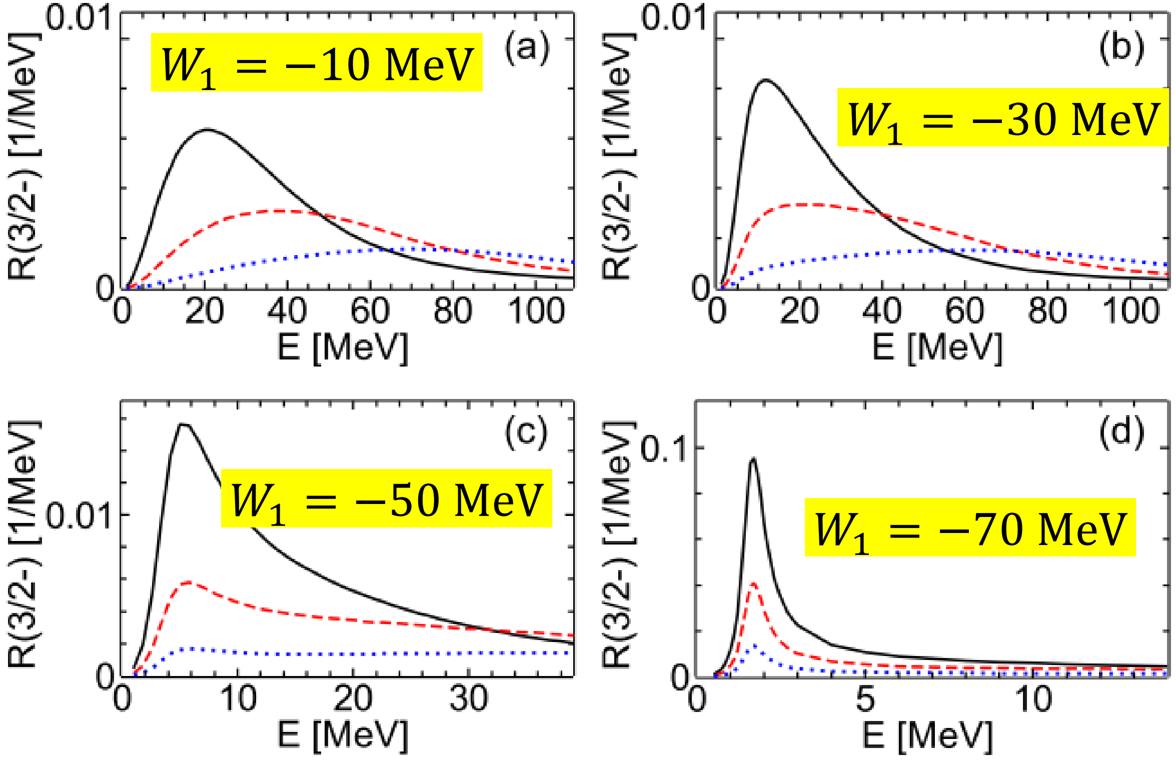
$-36 \sim -30$ MeV

RIKEN2016 experiment $^4\mathrm{He}(^8\mathrm{He}, ^8\mathrm{Be})$

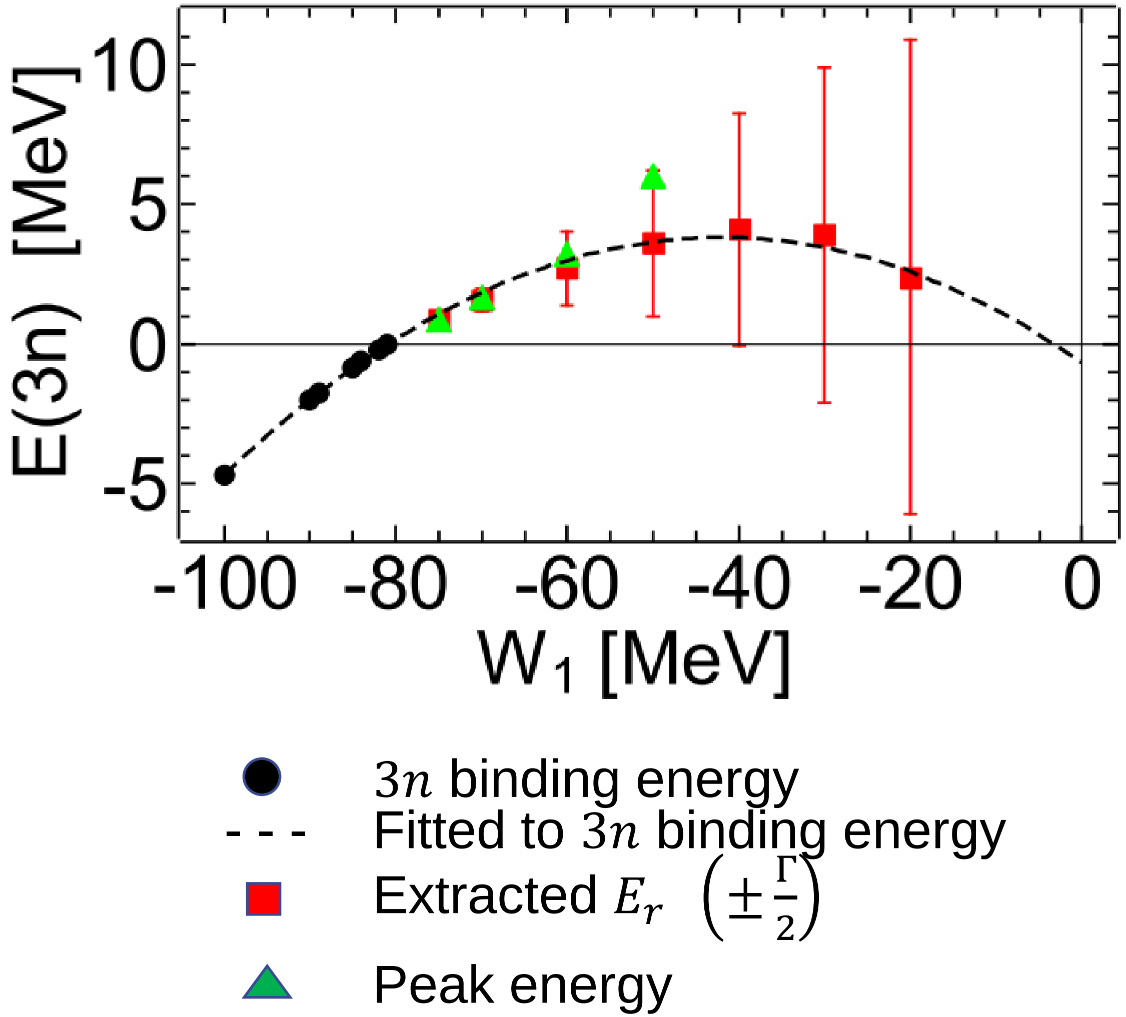
$\Leftrightarrow W_1 = -2.55$ MeV to reproduce ^{3}H binding energy

[2] Introducing a 3BP

$\boldsymbol{Q} = \mathbf{300}, \mathbf{400}, \textbf{and } \mathbf{500}\ \textbf{MeV/c}$



$[E_r \sim 4$ MeV, $\Gamma \sim 10$ MeV$]$ for $W_1 = -36$ MeV

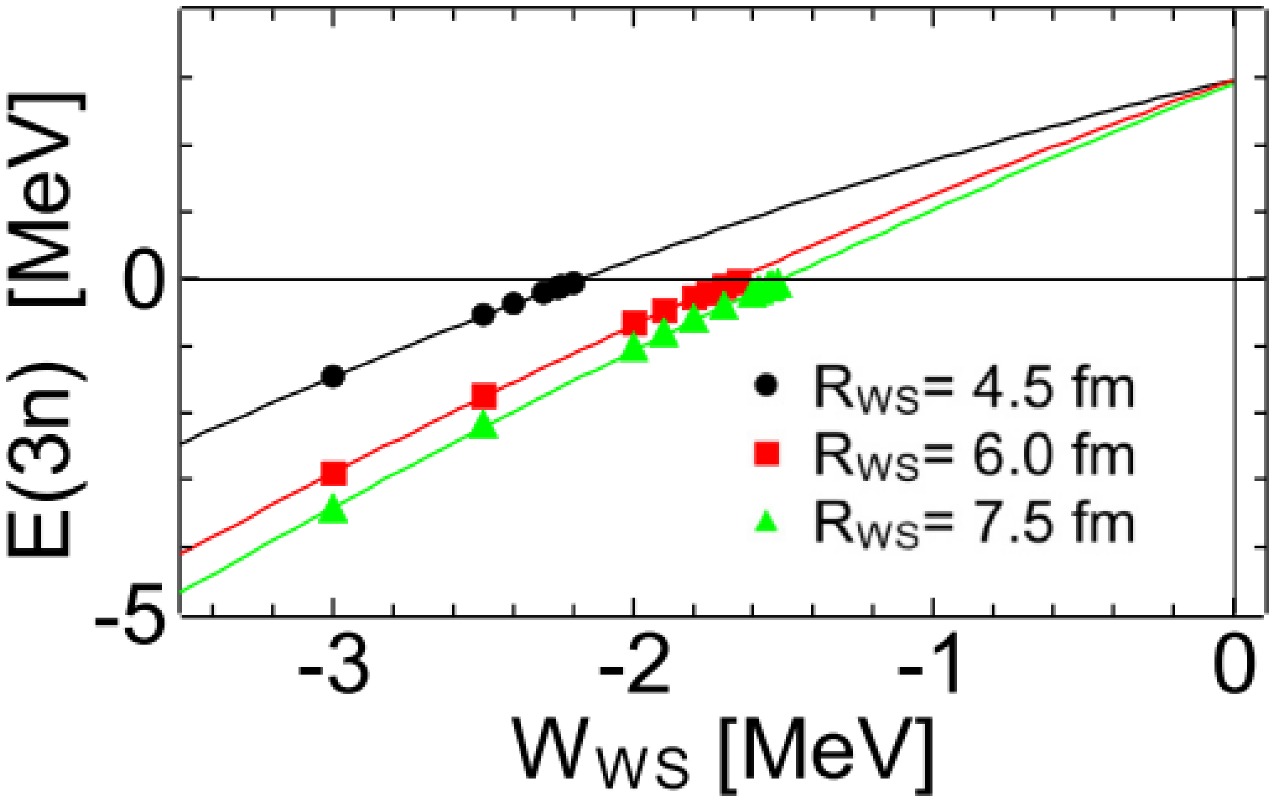


$W_1 \to 0$

No pole close to the real axis

[3] Additional trapping potential

$$W(r_i) = W_{\mathrm{WS}} \frac{1}{1+e^{(r_i - R_{\mathrm{WS}})/a_{\mathrm{WS}}}}, \qquad a_{\mathrm{WS}} = 0.65\ \mathrm{fm}$$



$3n$ resonance ?

Similar result with Gandolfi(2019) & Li(2019), which suggest the existence of 3n resonance.

[0] Calculations with Argonne V18$-nn$ potential

No resonance peak

Extrapolation methods

[1] Multiplying a factor to the nn potential

[2] Introducing a 3BP

Complex pole energy is far from real axis → nonexistence of $3n$ resonance

[3] Additional trapping potential

→existence of $3n$ resonance

?

“$2n$” system with Gaussian + trapping potential

- $3n\left(\frac{3}{2}^{-}\right)$ state ~ n-dineutron in P-wave (L = 1)

- 2-body (“$2n$”) P-wave state in trapping-potential

- Effective potential:

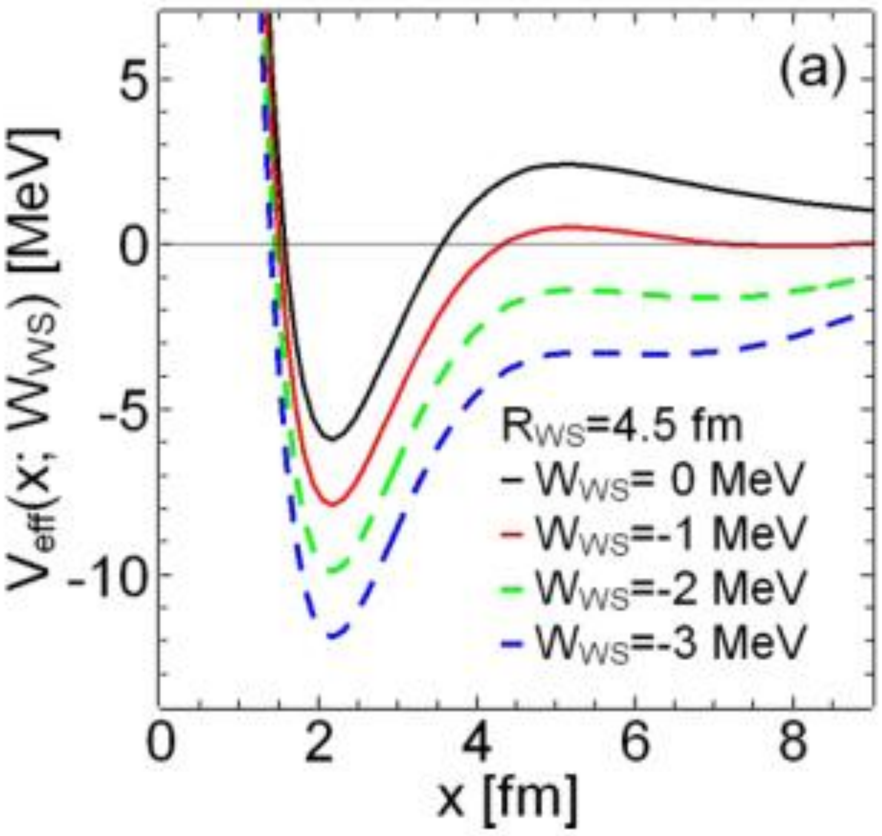
$$V_{\text{eff}}(x) = v_G e^{-\left(\frac{x}{r_G}\right)^2} + \frac{\hbar^2 L(L+1)}{m x^2} + \sum_{i=1,2} W(r_i)$$

Parameters: $r_G = 2.5\text{fm},\ v_G = -50\ \text{MeV}$ “no resonance state”

$$\mathrm{W}(r_i) = W_{\text{WS}} \frac{1}{1 + e^{(r_i - R_{\text{WS}})/a_{\text{WS}}}}, \qquad a_{\text{WS}} = 0.65\ \text{fm}$$

"$2n$" system with Gaussian + trapping potential

$$V_{\rm eff}(x) = v_G e^{-\left(\frac{x}{r_G}\right)^2} + \frac{\hbar^2 L(L+1)}{mx^2} + \sum_{i=1,2} W(r_i), \qquad L = 1$$



As the attractive effect is reduced, the barrier appears at positive energy.

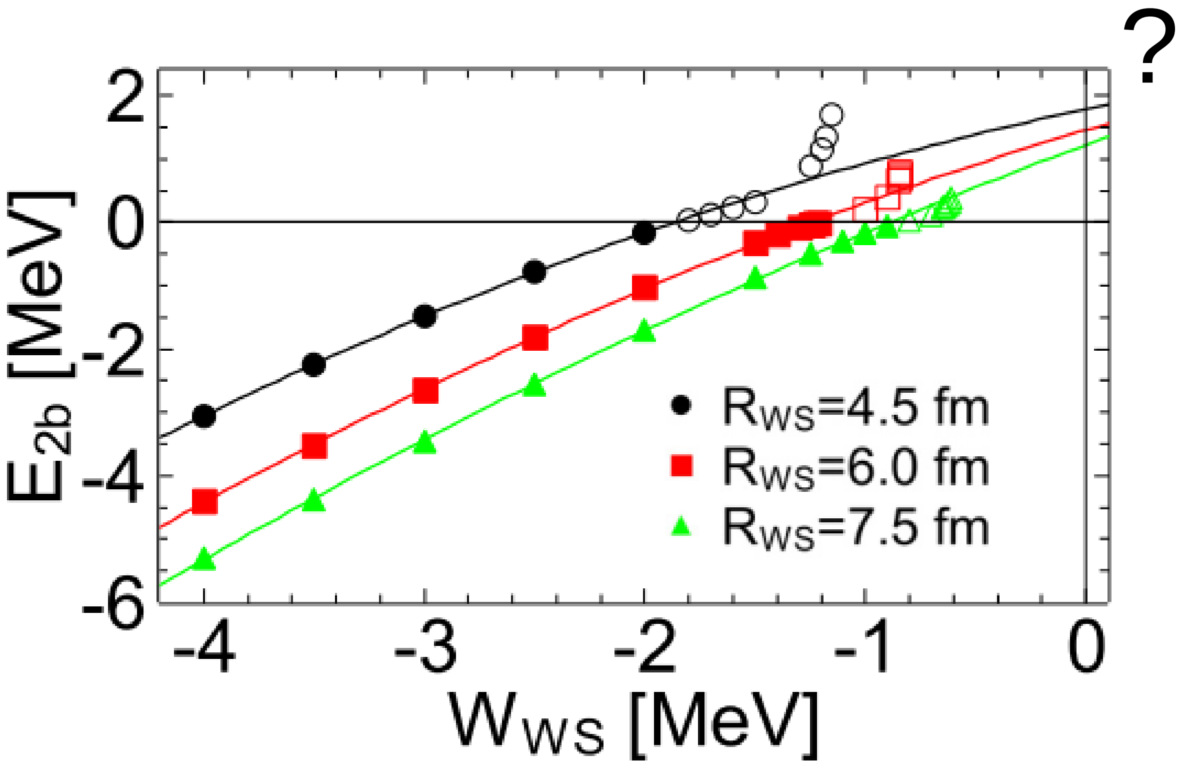
→

An extra repulsive effect that does not exist for the bound states.

solid curves → no bound state exists

dashed curves → a bound state

"$2n$" energies with trapping potential



Bound state
Resonance (phase shift=90°)

Extrapolation of bound state energies
→
Positive energy at $W_{\mathrm{WS}} = 0\ \mathrm{MeV}$

However, soon after getting into the continuum region, the W_{WS} dependence is quite different from that in the bound state region.

The extrapolation is no longer reliable.

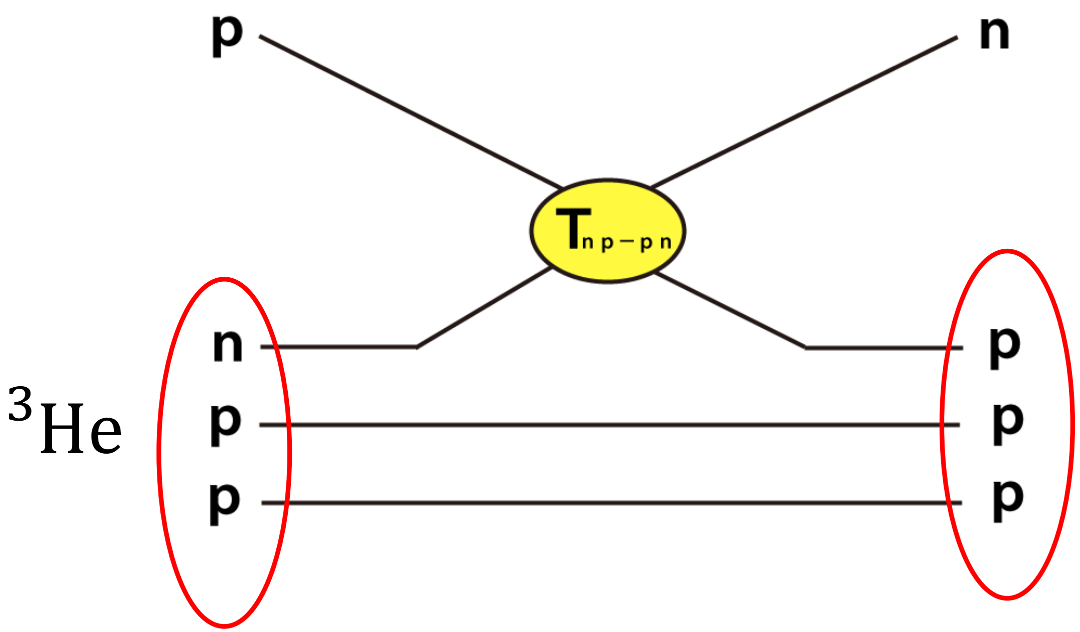
5. $^{3}\mathrm{He}(p,n)ppp$

PHYSICAL REVIEW C **77**, 054611 (2008)

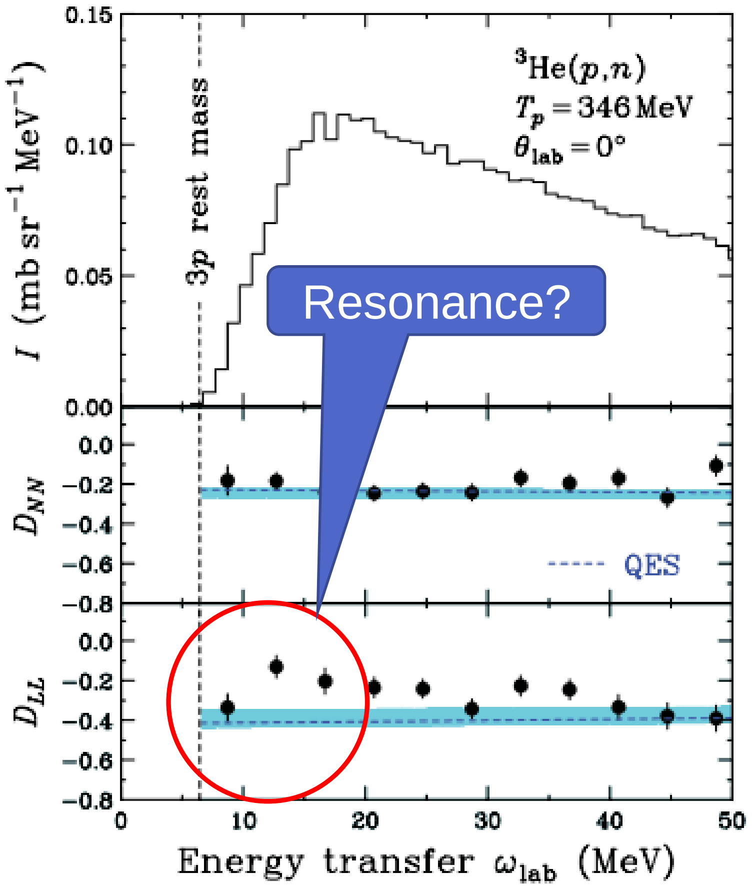
Complete set of polarization transfer coefficients for the ^{3}He(p, n) reaction at 346 MeV and 0 degrees

T. Wakasa,[1,*] E. Ihara,[1] M. Dozono,[1] K. Hatanaka,[2] T. Imamura,[1] M. Kato,[2] S. Kuroita,[1] H. Matsubara,[2] T. Noro,[1] H. Okamura,[2] K. Sagara,[1] Y. Sakemi,[3] K. Sekiguchi,[4] K. Suda,[2] T. Sueta,[1] Y. Tameshige,[2] A. Tamii,[2] H. Tanabe,[1] and Y. Yamada[1]

PWIA



$^3\mathrm{He}(p,n)ppp$



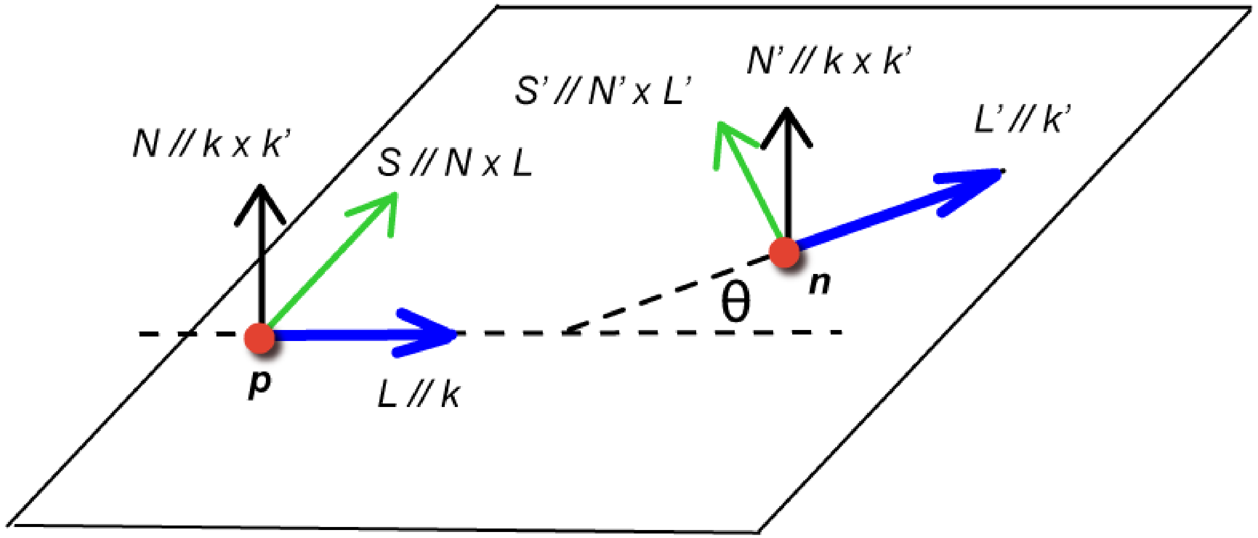
Horizontal lines:
$D_{NN}(0°), D_{LL}(0°)$ in p, n scattering

$$^3\mathrm{He}(p,n)ppp\ (\theta_n = 0°) \quad T_p = 346\ \mathrm{MeV}$$

$$\frac{d\sigma}{d\omega d\Omega}(0°), D_{NN}(0°), D_{LL}(0°)$$

$D_{LL}(0°)$?

$\omega_0 = 16 \pm 1$ MeV $\quad \Gamma = 11 \pm 3$ MeV



Response functions

- Spin-isospin response function for the transition process: $^3\mathrm{He} \to 3p$

$$R_C(E), R_L(E), R_T(E)$$

- $|\Phi_b\rangle$: $^3\mathrm{He}$ wave function

$$R_C(E) = \int dE' \sum_f \left| \left\langle \Psi_f(E') \right| \sum_i e^{i\vec{Q}\cdot\vec{r}_i} \tau_i^+ \left| \Phi_b \right\rangle \right|^2 \delta(E - E')$$

$$R_L(E) = \int dE' \sum_f \left| \left\langle \Psi_f(E') \right| \sum_i e^{i\vec{q}\cdot\vec{r}_i} (\hat{Q} \cdot \vec{\sigma}_i) \tau_i^+ \left| \Phi_b \right\rangle \right|^2 \delta(E - E')$$

$$R_T(E) = \int dE' \sum_f \left| \left\langle \Psi_f(E') \right| \sum_i e^{i\vec{q}\cdot\vec{r}_i} (\hat{Q} \times \vec{\sigma}_i) \tau_i^+ \left| \Phi_b \right\rangle \right|^2 \delta(E - E')$$

- Observables

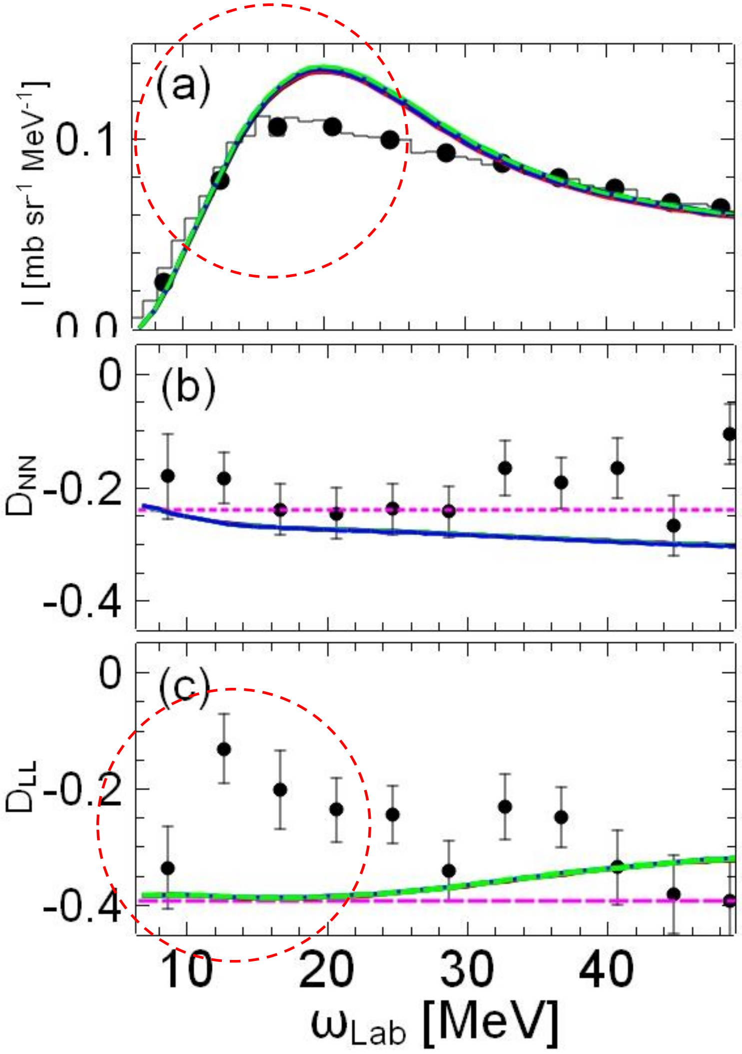
$$\sigma \propto |t_c(Q)|^2 R_C + |t_L(Q)|^2 R_L + 2|t_T(Q)|^2 R_T$$

$$D_{LL} = \frac{|t_c(Q)|^2 R_C + |t_L(Q)|^2 R_L - 2|t_T(Q)|^2 R_T}{|t_c(Q)|^2 R_C + |t_L(Q)|^2 R_L + 2|t_T(Q)|^2 R_T}$$

$$D_{TT} = \frac{|t_c(Q)|^2 R_C - |t_L(Q)|^2 R_L}{|t_c(Q)|^2 R_C + |t_L(Q)|^2 R_L + 2|t_T(Q)|^2 R_T}$$

$^3\text{He}(\vec{p},\vec{n})ppp \quad T_p = 346\text{MeV} \quad \theta_n = 0°$

NN-potentials: AV18, AV14, AV8', dTRS



Momentum transfer $Q \sim 10 - 50$ MeV/c

Scattering amplitude of $pn \to np$ [SAID, NN-online]

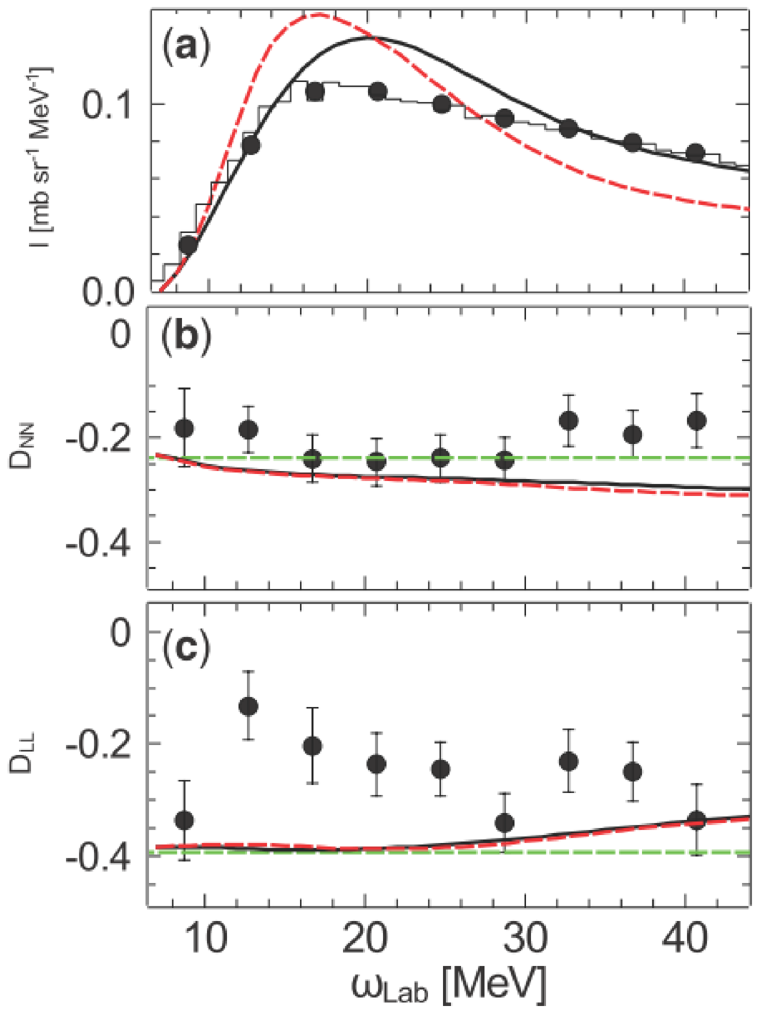
$$t(\vec{Q}) = t_c(Q) + t_L(Q)(\hat{Q}\cdot\vec{\sigma}^0)(\hat{Q}\cdot\vec{\sigma}_i) + t_T(Q)(\hat{Q}\times\vec{\sigma}^0)(\hat{Q}\times\vec{\sigma}_i)$$

NN-amplitude online database
SAID Program,
http://gwdac.phys.gwu.edu/

NN-OnLine
http://nn-online.org/

$^3\mathrm{He}(p,n)ppp \quad T_p = 346\mathrm{MeV} \quad \theta_n = 0°$

Only 2NF vs. 2NF+3NF（$\boldsymbol{W_1 = -36}$ MeV）



Required value of W_1 for $4n(0^+)$ state to bind:

$$W_1 = -36.14\ \mathrm{MeV}$$

Three-body potential

$$W(T) = \sum_{n=1}^{2} W_n e^{-(r_{12}^2 + r_{23}^2 + r_{31}^2)/b_n^2}\, \hat{P}(T)$$

6. Summary

- Three different extrapolating methods from $3n$ bound state energies to continuum states:
  (i) to enhance component of the nn potential [No 3n resonance state]
  (ii) to introduce a three-body force [No 3n resonance state]
  (iii) to add an external attractive trapping potential [3n resonance state]
- This discrepancy occurs due to the longer range trapping potential, which destroys the potential barrier.
- This defect occurs in general, and the trapping method should be used carefully in studies of resonance states of few- and many-body systems.
- Precise calculations for reactions to study 3n or 3p systems (e.g,. $^{3}\mathrm{He}(\vec{p},\vec{n})ppp$) are now available.